FOUNDATIONS OF REAL ESTATE FINANCIAL MODELLING

ROGER STAIGER
Foundations of Real Estate Financial Modelling

*Foundations of Real Estate Financial Modelling* is specifically designed to provide an overview of pro forma modelling for real estate projects. The book introduces students and professionals to the basics of real estate finance theory before providing a step-by-step guide for financial model construction using Excel. The idea that real estate is an asset with unique characteristics which can be transformed, both physically and financially, forms the basis of discussion.

Individual chapters are separated by functional unit and build upon themselves to include information on:

- Amortization
- Single-Family Unit
- Multifamily Unit
- Development/Construction Addition(s)
- Waterfall (Equity Bifurcation)
- Accounting Statements
- Additional Asset Classes.

Further chapters are dedicated to risk quantification and include scenario, stochastic, and Monte Carlo simulations, waterfalls, and securitized products. This book is the ideal companion to core real estate finance textbooks and will boost students’ Excel modelling skills before they enter the workplace. The book provides individuals with step-by-step instruction on how to construct a real estate financial model that is both scalable and modular.

A companion website provides the pro forma models to give readers a basic financial model for each asset class as well as methods to quantify performance and understand how and why each model is constructed and the best practices for repositioning these assets.

**Roger Staiger** (FRICS) is Managing Director for Stage Capital, LLC, a global advisory firm in real estate financial modelling, portfolio management and asset repositioning. He holds faculty positions at George Washington University, Georgetown University and Johns Hopkins University. He has held many senior positions including Managing Director for Constellation Energy’s Retail Commodity Division and CFO for America’s Best Builder 2006.

www.routledge.com/cw/staiger
Foundations of Real Estate Financial Modelling provides the reader with a clear path towards appropriate methods and processes used to properly model various types of real estate investments. In addition, the book provides the “why” behind the models by introducing practical explanations of valuation, capital stack formation and assessment of risk. The book also defines key terms, which are especially important in today’s real estate industry where jargon and undefined “rule of thumb” benchmarks proliferate.

What is noteworthy is Professor Staiger’s use of visual enhancements (graphs and actual pro-formas) throughout the book that bring clarity and meaning to the content. This is effective because many adult learners are visual in their comprehension of new material.

In summary, students will appreciate the thoughtfulness of the book that provides a roadmap to a clear and concise explanation of the process and practical application of real estate models for financial forecasting and risk assessment.

Bob Rajewski, Adjunct Professor, John Hopkins University

The book is aptly named. The author does an excellent job of providing the context in which real estate modelling should be viewed in. Rather than being a step-by-step instruction manual, Staiger reviews the appropriate supporting theory that must be understood before diving headlong into real estate financial modelling. As real estate is not a commodity, there is no “one size fits all” financial model. Given the extreme variations in assumptions, risk profiles, and deal structures, real estate financial models are as diverse as the assets they attempt to quantify. Failure to understand these concepts along with the industry standard metrics will almost guarantee the construction of flawed models.

In general terms, pro forma modelling is fairly straightforward with a basic understanding of finance. However, many seasoned analysts struggle with modelling waterfalls. As the waterfall is designed to separate risk and associated returns by tranches, it therefore requires the adjustment of payment streams. Done incorrectly the outcome can be disastrous causing potential investors to lose faith in the integrity of the model and possibly the entire project. The author devotes a substantial amount of time covering this topic and incorporates a variety of diagrams to simplify the concepts.

I recommend this book to anyone attempting to build a defensible real estate financial model. This book will inspire confidence to both beginner and advanced model builders.

Keith A. Hopkins, MBA, Managing Director, Stage Capital Group, LLC

Foundations of Real Estate Financial Modelling was created as part of Professor Staiger’s curriculum within the School of Real Estate at Georgetown University’s School of Continuing Studies. It is a superb step-by-step process of how to build various models which are both scalable and modular. The textbook will be an excellent resource for students, as well as non-finance faculty, to understand the mechanics of building a real estate pro forma.

Roger’s disciplined and purposeful approach to pro forma modelling has been so well received by the students that we regularly offer workshops, free to students, to ensure the skill is gained and focused upon in the program. This textbook is an excellent resource for the students to supplement the workshops and explain the basic theory of real estate finance coupled with technical skills.

We are proud that we will be incorporating this textbook within our curriculum at Georgetown University. This textbook could well be required reading for all students.

William H. Hudnut III, Executive Director, Georgetown University
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I would like to express my sincere gratitude to the many people who helped me author this book over the past two tumultuous years of my life – a period which saw my father die under suspicious circumstances and resulted in a will caveat. This ended with a unanimous jury verdict finding that his will was forged and not duly executed. (This will be the subject of a different book.)

On the darkest of days, this book was my lifeline to sanity and progression. The support over the past two years this book and I, in particular, have received has been tremendous. Bill Hudnut, at Georgetown, absolved me of all administrative responsibilities. Mike Anikeeff, at Johns Hopkins, ran interference with the administration, who always seems to trip up on themselves. Bob Rajewski, my respected friend and colleague, stood in whenever court appearances disallowed me to cover a specific class (I will never be able to thank you enough). Ed Harding, of Team13System, the smartest accountant/finance person I have ever known, edited the book, providing countless hours of improvement.

Two students authored chapters: Reagan Mosley and Michael Cardman, Pro Forma Portfolio Modelling and Structured Products, respectively. Your tireless hours and superb efforts are appreciated and noted. Reagan, I promise to never tease you in class about Tinder. Michael, your efforts as TA made it possible to write my book in the evenings. To both, thank you.

Finally, Jennifer, my secret weapon and the smartest person I have ever met, thank you. At the beginning of the court saga regarding my father, I put my hand in the air and said, “You always said you would help me if I needed it. Well, I need ‘help,’ please help me.” The tireless hours you spent focused on my court case which brought justice for my family will always be remembered. To all reading this, while Jennifer would never say anything because she is soft-spoken and polite, she attended Harvard ’91 and Yale Law ’95. For all your love and assistance, I promise to never question your degree in women’s studies at Harvard (a dual major actually with economics) or ask why you cannot iron a shirt when that should have been covered by the end of your first year at Harvard!

Although I have mentioned a few people here, the actual list is long and unending. From students encouraging me to finally publish to peers anxious to see the long-awaited book, I thank you for your consistent and constant encouragement. While my name may be on the cover (which I love, by the way), I fully recognize it has been a long journey, and please know I recognize everyone’s efforts and readily admit this was far from a singular effort.

Thank you!
Preface

*Foundations of Real Estate Financial Modelling* is a book designed to assist individuals in developing real estate pro forma models beginning with a fresh spreadsheet. Although the first three chapters cover theory and provide background for real estate, the book is actually a “how-to” book on pro forma construction.

In the first three chapters readers review the basics of real estate, valuation, and cash flow distinctions between asset types (i.e. hotel, multifamily, retail, etc.). In addition, readers are introduced to a new metric, P(Loss), developed by the author. P(Loss) quantifies the probability of not returning 100.0% of invested capital, that is, the most important question for an investment. It is innovative in its simplicity and requirement of risk quantification.

In later chapters, beginning with Chapter 4, readers follow the chapters to construct ever-more complex real estate financial models. The finished versions can be found on the companion website for use after the reader has personally completed the model. What the book does not do is analyze the use of these models, though real case studies are included on the companion website. The book focuses on construction rather than use. This book is best used as a classroom tool or by the novice to intermediate financial analyst who has a background in real estate.

Later books will include more analysis of case studies using the base models constructed within this text. For now the focus is on model construction rather than case study analysis.
When considering the question “What is real estate?” more questions are produced than answers. Real estate is considered to be real property. This translates the question to what is “property”? For the purposes of this book, real estate is defined as investment(s) in physical buildings and land, in both combined, or in financial products backed by physical buildings and land, such as mortgage-backed securities (MBS). Therefore, the mental image of real estate is generally a building, home, or farm – i.e. a physical structure residing on land. However, the picture of real estate should actually be a return distribution which includes both return and risk. Real estate is tangible, it can be seen, touched, felt, and at times of distress, kicked. Real estate is also intangible as it is a financial instrument which can be transformed physically and financially. Finally, the value of real estate, in reality, is the cash flow it produces, which is a direct result of the tenants and leases and the risks associated with these cash flows. Conceptually, the physical real estate is an expense item while the tenants produce the cash flows for a real estate project. The characteristics of these cash flows depend on the type of real estate (i.e. asset class) and the current economic environment.

To consider real estate in a broader context, it is an asset class which attracts investment funds. In fact, real estate is the second largest asset class in the US and was actually the largest asset class in 2006, prior to the bubble bursting. To provide some context, in 2010 the US GDP was approximately $14,500bn, with total US asset values (aggregation of Fixed Income, Equity, and Real Estate) of approximately $69,000bn. In 2010, the asset split was Fixed Income ($32,500bn), Equity ($18,000bn), and Real Estate ($18,500bn). Therefore, real estate, in 2010, was approximately 27% greater than US GDP and the same percentage of US asset value.

Real estate in the context as an asset class must be bifurcated further into residential and commercial components. Residential is four times the size of the commercial asset class and is actually a leading indicator by 7 months (based on historic analysis of the Case-Shiller index versus RCA’s commercial index in 2014). Residential real estate is therefore approximately $14,800bn while commercial real estate is $3,700bn (note: commercial real estate in 2007 was $6,000bn).

Although the differences between residential and commercial may appear to be obvious, it is important to reclassify each to best describe their financial characteristics – i.e. residential is non-income producing and commercial is income producing. Classifying commercial as income producing is the conventional methodology. However, the rhetorical question remains, “Are both real estate asset classes not both non-income producing?” The difference is that the commercial asset class has historically been purchased for investment purposes (i.e. an investment-type asset), while traditionally residential real estate purchased as a primary home is considered for personal consumption (i.e. residential home). To further

1 What is real estate?
extrapolate, residential is also quasi-income producing when one considers the rental expense that would be incurred if individuals were not in their own homes. MBAs call this deference of rental expense “opportunity cost” and model it as if it were an expense to determine price.

At the core, all real estate has the same cash flow diagram (Figure 1.1).

Specifically, real estate is unique as an asset class because, unlike equity and fixed-income securities, the purchase is rewarded not with income (e.g. dividends and/or coupon payments) but rather with invoices. These invoices are maintenance, in the case of a physical structure, property taxes, etc. Basically the ‘rewards’ for owning real estate, in its purest form, are expenses which are both timely and constant. Of course the owner also has use of the property and owner’s rights to the property, but it is the tenants that provide the income for the asset. Further, these expenses are known and can be accurately forecast given historic depreciation tables and tax forecasts. Therefore, the value of real estate, in purity, is negative.

Then why does real estate trade with positive pricing? The answer: the ability of real estate to attract cash flows, realized or not. Of course there are other reasons as well – for example the ability to absorb large amounts of capital in singular transactions, community investment and presence, and so forth – but in summary, it is the ability to actively manage the asset to produce positive cash flows (i.e. income over maintenance, taxes, and debt service). Therefore, the value of real estate is not the physical structure itself but rather the rents, the leases, the asset can attract.

This is the subject of Chapter 12, but the value of real estate can be modelled using Portfolio Theory. In the case of an office project where multiple tenants occupy the asset, the value is determined as the portfolio of leases rather than as a single project, or office asset. This addresses repositioning given that a single-tenanted building provides no diversification of industry and risk while a multi-tenanted building provides diversification, provided the tenant’s industries are not highly correlated. Extrapolating further, a multi-tenanted real estate asset can be modelled as and considered a portfolio of forward rate agreements (FRAs) rather than a single asset. Again, this is the subject of later chapters.

An actively managed real estate investment with tenants will have the more traditional cash flow diagram associated with equity and fixed-income securities (Figure 1.2).

Note that the previous diagram assumes a terminal value at the end of the period. There are two methods of valuing a real estate asset: (1) assume cash flows continue into perpetuity and (2) assume cash flows end upon sale, or terminal value. For the purposes of this text, a terminal value will be assumed in calculating yield; however, it should be understood that
the terminal value can be removed if the property is considered held (i.e. valued) into perpetuity (note: terminal value in this text will be quantified using the traditional capitalization rate method). See Figure 1.3.

Note that sales price is calculated as a function of net operating income (NOI) rather than net income. Net income is NOI less debt service, that is, cost of financing. This is consistent with Modigliani and Miller’s propositions from corporate finance, which state the value of an entity is independent of financing concerns.

**Yield**

Yield is a valuation method used for Western finance – i.e. Europe, North and South America. It is the solved discount rate for the series of cash flows when setting the Net Present Value (NPV) equal to zero, in other words, the Internal Rate of Return (IRR). This is more simply viewed in Figure 1.4 and Figure 1.5.

The NPV equation is a polynomial. Therefore, it is important, to avoid multiple IRR possibilities, that there be only a single cash flow change – i.e. cash flows change from negative to positive once. If there are multiple changes in cash flow signs, as in the following example, there is the possibility of multiple IRR solutions.

Note: there are other pitfalls when completing an IRR; however, they are not all covered in this text. Some of these pitfalls include type of project consideration (i.e. financing versus investment) and using IRR to compare multiple projects with different initial capital and varying time lines.
4 What is real estate?

Net Present Value (NPV)/Internal Rate of Return (IRR) in detail

Net Present Value (NPV) is the Present Value (PV) of all future cash flows, discounted at the appropriate market rate or the rate of alternative investments, minus the initial cash outlay. The Net Present Value rule is that an investment is worth considering when the NPV is a positive. It is calculated using the formula in Figure 1.6:

\[ NPV = C_o + \sum_{i=1}^{N} \frac{C_i}{(1 + rate)^i} \]

*Figure 1.6 Net Present Value Equation (Rearranged)*

where
- \( C_o \) = Initial Cash Outflow
- \( i \) = period or timing of cash flow
- \( rate \) = discount rate used to evaluate the cash flows

Without exception, NPV can be used to determine value of projects to the equity holders. Accepting projects with a positive NPV always benefits the equity holders, assuming an appropriate discount rate has been applied.

Internal Rate of Return is the rate calculated by setting the NPV equation to zero for a series of cash flows. It is the premier method for evaluating capital expenditures. It provides an intrinsic value of a project, expressing the realized rate of return for future cash flows. However, unlike NPV, IRR has pitfalls and does not always determine which cash flows offer positive value to equity holders unless it is completely understood.

The following is a list of some, but not all, of the pitfalls to using IRR for cash flow evaluation.

- Multiple IRRs
- Investment versus financing
- Scaling (specific to mutually exclusive projects).

The first, multiple IRRs, is important since the NPV formula is a polynomial when solving for the rate. Therefore, for specific cash flows, those flipping from negative to positive or positive to negative more than once, multiple IRRs may be found. The following example illustrates a series of cash flows, with six flips in sign, in which two IRRs exist. Neither is correct and neither is incorrect. It is impossible to evaluate these cash flows using the IRR method. See Figure 1.7.

When graphing the NPV versus rate, two IRRs are located at each point of an X-intercept (3% and 58%). Both are correct and neither offers a true representation of the worth of the cash flows to equity holders. See Figure 1.8.

<table>
<thead>
<tr>
<th>Date</th>
<th>1/1/90</th>
<th>1/1/91</th>
<th>1/1/92</th>
<th>1/1/93</th>
<th>1/1/94</th>
<th>1/1/95</th>
<th>1/1/96</th>
<th>1/1/97</th>
<th>1/1/98</th>
<th>1/1/99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-4,500</td>
<td>4,000</td>
<td>3,000</td>
<td>4,000</td>
<td>-3,000</td>
<td>4,000</td>
<td>-1,000</td>
<td>3,000</td>
<td>-4,000</td>
<td>-6,500</td>
</tr>
</tbody>
</table>

*Figure 1.7 Cash Flow Example*
What is real estate?

The Net Present Value, discounted at 10%, yields a positive value of $1,408. Since the NPV is positive, the cash flows yield value to the equity holders. This would not be the conclusion if the IRR approach were used since it could yield a value of 3%, which is below the discount rate of 10%. Therefore, an NPV analysis yields the correct conclusion.

The second problem with IRR analysis is the altering valuation criteria for investment versus financing projects. Consider the following two projects, A and B, and their respective cash flows:

<table>
<thead>
<tr>
<th>Project</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cash</td>
<td>–$100</td>
<td>$100</td>
</tr>
<tr>
<td>Cash @ T = 1</td>
<td>$130</td>
<td>–$130</td>
</tr>
</tbody>
</table>

The Internal Rate of Return for each project is 30%; however, the NPVs for projects A and B are positive ($18.20) and negative (−$18.20), respectively, at a 10% discount rate. The IRR analysis would seem to state that both projects are of equal value but the NPV analysis clearly demonstrates that only project A adds value to the equity holders.

The difference is that project A is an investment project whereas project B is a financing project. An example of a financing project is a seminar where the money is received months in advance of the outlays, which occur at the time of the seminar. The rule for acceptance under the IRR criteria is to accept if the IRR is greater than the discount rate for investment-type projects and accept if the IRR is less than the discount rate for financing-type projects. See the graphs in Figures 1.9 and 1.10. This rule change for type of project is confusing and nonexistent in the NPV analysis.

A third pitfall for IRR analysis concerns mutually exclusive projects. Scaling is a critical issue when two opportunities are available but only one can be executed. An example is the
ability to earn $50 on one investment and $100 on another. Consider the following two projects, A and B, and their respective cash flows:

<table>
<thead>
<tr>
<th>Project</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cash</td>
<td>−$50</td>
<td>−$1000</td>
</tr>
<tr>
<td>Cash @ T = 1</td>
<td>$100</td>
<td>$1100</td>
</tr>
</tbody>
</table>

For this example, assume the period for the investment to be short, virtually instantaneous, so that timing effects can be ignored. The NPV for project A is $50 and for project B is $100. The IRR for project A is 100% and for project B is 10%. Using IRR analysis project A should be pursued since it yields the highest return to equity holders; however, NPV analysis shows that project B yields the highest value to equity holders. The IRR analysis is flawed because it neglects to account for the scaling issue that $100 is worth more to the equity holders than $50. This can be adjusted using an incremental IRR analysis but that is beyond the scope of this book.

The three pitfalls mentioned here demonstrate the limitations of using the IRR analysis to evaluate cash flows to equity holders. IRR is beneficial in stating the intrinsic value of future cash flows but it must be completely understood. NPV analysis always yields correct
conclusions but does not state the intrinsic value. Both methods should be used together to ensure accurate valuation of cash flows.

Modified Internal Rate of Return (MIRR)

Modified Internal Rate of Return (MIRR) adjusts for the pitfall of traditional IRR analysis which assumes all cash flows are reinvested at the calculated IRR rate. Therefore, traditional IRR analysis may misstate the implicit return for a project by failing to quantify the effect that earned cash flows during the project are not reinvested at a project’s IRR but rather at a corporate reinvestment rate (estimated at the corporate weighted average cost of capital, or WACC).

Modified IRR corrects this misstatement by converting a project’s cash flows to a zero coupon security. The project’s future cash flows are compounded to the final period at the reinvestment (WACC) rate. Using the initial cash outflow, the yield on the zero coupon security is then calculated (MIRR). The equation for MIRR is in Figure 1.11.

\[
MIRR = \left[ \frac{\sum_{s=1}^{n} (CF_s)(1+r)^{n-s}}{CF_0} \right]^{1/n} - 1
\]

where

- \( CF_s \) = Cash Flows in period S
- \( CF_0 \) = Initial Cash Flow (Cost)
- \( n \) = Number of periods
- \( r \) = Reinvestment rate
- \( S \) = Current period

An example of the Modified Internal Rate of Return for a series of cash flows follows in Figure 1.12:

![Figure 1.12 Future Valuing Cash Flows for MIRR](image-url)
What is real estate?

The compounding of the cash flows creates a zero coupon security maturing at time period \( n \) (Figure 1.13).

\[
\sum_{s=1}^{n} (CF_s)(1+r)^{n-s}
\]

Figure 1.13 MIRR Zero Coupon Bond

Note the Price \( (CF_0) \) for the zero coupon security is as follows in Figure 1.14:

\[
P = \frac{M}{(1 + yield)^n}
\]

where

\[
M = \sum_{s=1}^{n} (CF_s)(1+r)^{n-s}
\]

\( yield = \text{MIRR} \)

\( n = \text{Periods} \)

Figure 1.14 Price for MIRR

Solving for MIRR yields the equation in Figure 1.15:

\[
MIRR = \left[ \frac{M}{P} \right]^{\frac{1}{n}} - 1
\]

Figure 1.15 MIRR Equation

Risk

‘Risk’ is understood largely by everyone but the true definition seems to be elusive. Before a discussion of risk commences, risk must be bifurcated into project risk (i.e. single entity) and portfolio risk (i.e. multiple assets held together). Single entity risk, which will be discussed here, quantifies deviation of an expected return for a single project; for example
project A had an expected return of 18% with a standard deviation of 5%. Single entity risk assumes a project is held in a vacuum and does not consider additional assets held together. Project risk assumes assets are held together and viewed in their entirety. Portfolio risk therefore not only considers individual project return and risk but also the correlation (i.e. linear association of projects), as well as the respective weights of each asset held in the portfolio. For discussion purposes, project risk will be discussed early in this book whereas portfolio risk will be discussed in later chapters. Therefore, the discussion that follows is for single entities only.

The actual definition of risk is simple: the deviation or variation from an expected outcome. Basically risk is the range of outcomes from the expected value. If one assumes the distribution type is normal, risk, in its most basic form, is the deviation (standard deviation) of a normal curve (Figure 1.16).

![Figure 1.16 Normal Distribution Graphic](image)

It is essential to understand that while this drawing of risk is a representation, it does not depict all representations of risk. Fundamentally, risk is simply deviation from an expected return. This deviation is often represented by a normal distribution, as humanistic and nature data are best represented as normal distributions. However, risk can be modelled using a myriad of distribution types to include hypergeometric, uniform, triangular, Poisson, etc. The family of distributions includes those both continuous and discrete. The actual characteristics of each should be understood; if they are not, please reference a basic statistics text. For the purpose of this text about asset reposition, the normal distribution, which is a continuous distribution, will be used as a proxy for risk deviation and quantification. Note: there are two general types of distributions, continuous and discrete. Continuous distributions have no probability associated with a singular point but only within a range of points, such as between two points.

There are three main methods to quantify a project’s risk: (1) variance, (2) standard deviation, and (3) range. Understanding that the three are all related is essential. Further, each can be used as an approximation for another quantitative measure through the use and understanding of the relationship in Figure 1.17:

\[ \sigma = \sqrt{\sigma^2} \approx \frac{\text{Range}}{6} \]

![Figure 1.17 Risk Relationships](image)

The approximation assumes the distribution is best described by a normal distribution and assumes no outliers in the distribution (Figure 1.18); that is, the empirical rule states +/- 3\(\sigma\) represents approximately 99.7% of the data. Outliers are defined as data values falling outside this range.
Understanding the approximation is critical for a simple calculation of risk for projects. Note that a more detailed quantification for risk must use a stochastic approach (such as Monte Carlo) and is beyond the discussion for this text. The simple calculation for risk of a project uses the best-case IRR (yield) and the worst-case IRR. The range is calculated by taking the delta (Figure 1.19):

\[
IRR_{\text{Best Case}} - IRR_{\text{Worst Case}} = \text{Range}
\]

Figure 1.19 Range

The best case will be the highest IRR possible for the project with all ‘bull’ projections: highest rents, lowest vacancies, etc. The worst case is the polar opposite for the project with all ‘bear’ projections: lowest rents, highest vacancies, etc. For example, if the best- and worst-case IRR were 50% and −10%, respectively, then the range would be the delta between them. For example, 50% − (10%) = 60% (note: parentheses indicate negative value). Dividing the range by 6 yields a result of 10%. Therefore, the standard deviation – risk – of this project, assuming distribution is normal, is 10% (i.e. \( \sigma \approx 10\% \)); see Figure 1.20.

\[
\frac{\text{Range}}{6} = \text{Risk}(\sigma)
\]

Figure 1.20 Risk Approximation

Scenario risk quantification

Although range divided by 6 is a simple calculation, it can be misleading because it assumes the underlying distribution is normal. For real estate this is often not the case. While still a relatively simplistic risk quantification, scenario analysis is superior as it allows for a
nonsymmetrical distribution to be approximated. The basic equation for scenario risk (variance) quantification is in Figure 1.21.

This equation may appear new to many, but it is actually the broader definition of variance than what was provided in lower-level education. The two equations most often demonstrated are sample and population variance. The difference is $P_i$, which allows for non-equal probabilities for each event, $R_i$.

For example, the two equations for sample and population variance follow in Figure 1.22.

In both cases, population and sample, the probability of each event $R_i$ is equally probabilistic. The reason for the difference between population and sample (i.e. the minus 1) is that the sample uses $x$-bar to estimate the population parameter $\mu$. This approximation requires a degree of freedom loss adjustment in the denominator (i.e. $n-1$).

The scenario risk (variance) equation is therefore most applicable to generic cases where the probability of each event is not equal. As a demonstration, assume a real estate project has the following returns and probabilities of events (Figure 1.23). Note that event probabilities are difficult to quantify and often are utilized from past experiences on similar projects or projections based on known events going forward. The return calculations for the real estate project were estimated using Internal Rate of Return (IRR); however, the return could have been calculated utilizing numerous methods: MIRR, average Cash-on-Cash, etc.

$$\sigma^2 = \sum_{i=1}^{n} [E(r_i) - E(r)]^2 p_i$$

*Figure 1.21 Scenario Risk Equation*

$$\sigma^2_{\text{Population}} = \sum_{i=1}^{n} \frac{(R_i - \mu)^2}{n} \text{ and } \sigma^2_{\text{Sample}} = \sum_{i=1}^{n} \frac{(R_i - \bar{x})^2}{n-1}$$

where

$$P_{\text{Population}} = \frac{1}{n} \text{ and } P_{\text{Sample}} = \frac{1}{n-1}$$

*Figure 1.22 Population and Sample Standard Deviation Equations*

<table>
<thead>
<tr>
<th>$E(R_i)$</th>
<th>$P(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 40.0%</td>
<td>10%</td>
</tr>
<tr>
<td>2) 10.0%</td>
<td>60%</td>
</tr>
<tr>
<td>3) -20.0%</td>
<td>30%</td>
</tr>
</tbody>
</table>

$E(R) = (0.4 \times 0.1) + (0.1 \times 0.6) + (-0.2 \times 0.3) = 0.04 + 0.06 - 0.06 = 0.04$

$\sigma^2 = (0.4 - 0.04)^2 (0.1) + (0.1 - 0.04)^2 (0.6) + (-0.2 - 0.04)^2 (0.3) = 0.013 + 0.0022 + 0.0173$

$\sigma^2 = 0.0173 \therefore \sigma = \sqrt{0.0173} = 0.1315$

*Figure 1.23 Project Return and Risk Example*
What is real estate?

The project parameters are therefore:

\((4.00\%, 13.5\%)\)

Note that had each event been assumed equally probabilistic, the expected return, \(E(R)\), that is, the central location point, would have been 10% and the risk 10%. This would result in significant overestimation of expected return and underestimation of risk.

Further note: if the assumption remains that the distribution is normal, the empirical rule can be used to determine the distribution’s characteristics. If a normal distribution is not appropriate, Chebychev’s formula must be employed.

The important point here is to recognize projects have both an expected return (mean) and risk (standard deviation). Without the understanding of both for a project the most basic question of investing cannot be addressed, that is, what is the probability one will lose money.

Distribution shape(s)

Returning to the definition of real estate with an understanding of return and risk, a visual best describes the asset class. When one considers real estate in the context of both equities and fixed-income securities, the unique characteristics are exemplified. For instance, a direct investment in physical real estate usually involves leverage (i.e. borrowed capital). A typical million-dollar purchase will have the traditional 80% debt and 20% equity capital stack.

The 20% equity is the ‘down payment’ and represents capital injected by the owner. The 80% is borrowed funds from an external source, generally a lender. Further, the 80% debt may require an external guarantee from the owner. Finally, the ownership of the physical real estate also exposes an owner to the liabilities associated with the real estate, for example cleaning toxic, environmentally unfriendly soils. Therefore, a physical real estate purchase may expose the owner to considerably more financial risk than the initial equity commitment or even the entire capital stack. The total loss in a physical real estate investment may be unlimited. For example, a $10m property is leveraged 80%. If the property just disappears (not realistic but this is theoretical), the $8m debt is still owed. Now, if the property has disappeared and the soil is toxic and must be remediated, this is the responsibility of the owner and therefore poses risk greater than the asset’s original price.

For the moment, as a point of contrast, the return distribution for equity and fixed-income security (Figure 1.24) limits the loss of investment to 100.0%. In statistics parlance, this is similar to a lognormal distribution where the loss is anchored at 100.0% and the gain trails to positive infinity. With the rare exception of purchasing an equity below par (the details of this are beyond the scope of this text), the maximum loss for an equity purchase is the total invested capital. The same is true for a fixed-income security. The maximum loss for a fixed-income purchase is the total invested capital as well.

Returning to real estate, as discussed earlier, the maximum loss, in theory, is negative infinity (Figure 1.25). This is largely due to the unlimited liability associated with the

![Figure 1.24 Equity/Fixed-Income Return Distribution](image-url)
purchase of an asset. While this can be mitigated through legal maneuvering (i.e. purchasing the asset in an LLC, bankruptcy, etc.), the risk of an enormous loss remains real. This risk is real in real estate, as the 2007–08 Great Recession demonstrated. Using the Case-Shiller pricing index as a guide to US residential real estate, peak-to-trough differences were over 35%. Therefore, a $500,000 home with 80% leverage, purchased at the peak of the market and sold at the trough, using simple math, required a $75,000 check to be written at time of sale to satisfy the lender (ignoring principal pay down and the time period between peak and trough, which was about 2 years).

Efficiency/probability of loss (P(Loss))

How is a project best described? Most often a project is described by the yield alone. This characterization is not only immature but also dangerous. Yield fails to answer the most basic question of investment: will I get my money back? Stated differently, yield fails to answer the question pertaining to Return of Capital. The question that yield does address is how much money will the project earn (i.e. yield) if all assumptions and forecasts prove to be correct. Yield is therefore, fundamentally, Return on Capital.

To summarize, the three fundamental goals of investing are as follows, in order of importance.

1. Stay out of jail
2. Return of Capital
3. Return on Capital.

For the purposes of this text, only numbers 2 and 3 will be covered. Legal structures and issues with real estate ownership are left to texts which specialize and focus on legal issues. However, it is important to Question: note that earning money is actually the tertiary goal of investing and the least important.

To answer both the second and third goal for investing in real estate, a project’s characteristics must summarize both return and risk, that is, \((\bar{r}, \sigma)\), where \(\bar{r}\) is the project expected return. It is important to extrapolate both of these metrics and value projects, even though they are singular, as a portfolio consisting of one asset. Therefore, from Portfolio Theory, it is understood that the goal is not to maximize return but rather to maximize the efficiency. The equivalent is true for an individual asset held in isolation: the goal is to maximize the efficiency of the project.

What is efficiency? Loosely defined, efficiency is the point of maximum return for any given level of risk, in other words, the ratio of risk and return. A complete discussion of efficiency utilizing Portfolio Theory is beyond the scope and purpose of this text but for the
knowledge-seeking reader, the Markowitz Portfolio Theory should be referenced for a greater understanding.

How is efficiency quantified? There are three main methods to quantify efficiency: (1) Coefficient of Variation, (2) Sharpe Ratio, and (3) Treynor Ratio. This text will focus exclusively on the Coefficient of Variation metric. The Coefficient of Variation (CV) quantifies efficiency as in Figure 1.26.

Note that risk is in the numerator and return is in the denominator. Though it may at first be counterintuitive, maximizing efficiency is therefore minimizing the Coefficient of Variation.

Next, how does the CV respond to the goal for investing of Return of Capital? Again, we start with the assumption that project yield follows a normal distribution. Further, we remember from our Statistics 101 classes that a normal distribution is defined by two parameters: (1) central location and (2) standard deviation. Both values presumably have been quantified for the project.

It must also be understood that to quantify the Return of Capital, a project which has an expected return and standard deviation (risk) not equal to 0 and 1, respectively, must be translated to the standard normal distribution. A z-score is a method to translate a nonstandard normal distribution to the standard normal distribution, which has a central location (expected return) of 0 and a standard deviation of 1. The z-score equation is seen in Figure 1.27.

For the purposes of analysis, the z-score (which in Statistics 101 was an abstract formula to be used on an examination as a means to an ‘A’ grade) is now a major contributor to the understanding of risk for the sophisticated and the layperson. When using the z-score to translate a project’s performance, x-bar is the E(R) and σ the project’s risk. Xᵢ is the data value whose location is being transformed from a nonstandard normal curve via the z-score to the standard normal curve.

When analyzing Xᵢ = 0, it is at this point that the project actually loses capital, that is, returns less than the initial invested capital. Therefore, a z-score using Xᵢ = 0 represents the point at which a project loses money; stated differently, the area to the left of the z-score is the P(Loss) of the project. Demonstrated more simply, setting Xᵢ = 0 in the z-score simplifies to the image in Figure 1.28.

\[ CV = \frac{\sigma}{\bar{x}} \]

*Figure 1.26 Coefficient of Variation Equation*

\[ Z - \text{score} = \frac{-\bar{x}}{\sigma} \]

*where*

\[ x_i = 0 \]

*Figure 1.27 Z-score*

\[ Z - \text{score} = \frac{-\bar{x}}{\sigma} \]

*where*

\[ x_i = 0 \]

*Figure 1.28 Z-score (cont’d)
The next step is recognizing the relationship between this special case of z-score and the Coefficient of Variation (CV). Notice that the z-score is the negative reciprocal of the CV. Therefore, the area to the left of the z-score as calculated by the negative reciprocal of the CV is P(Loss). Remember that P(Loss) addresses the first basic question in investments (aside from legality), Return of Capital. (Note: P(Gain) = 1 − P(Loss).) See Figure 1.29.

An example of this translation is the 15-year historical return of the Case-Shiller index, composite-10 MSA. The historical year-over-year average return as calculated on a monthly basis is 5.38%, and the risk (i.e. standard deviation of returns) is 10.28% for the data ending July 2010. The results are summarized as such:

\[ \bar{x} = 5.38\% \]
\[ \sigma = 10.28\% \]

Although the data does respond to the concern of Return on Capital in the value 5.38%, it does not, untransformed, address the Return of Capital. To do this the Coefficient of Variation, efficiency, must be calculated. Once the CV is quantified, it can be transformed through the use of the z-score to calculate P(Loss), as in Figure 1.30.

\[ CV = \frac{\sigma}{\bar{x}} \therefore -\frac{1}{CV} = -\frac{1}{\frac{\sigma}{\bar{x}}} = -\frac{\bar{x}}{\sigma} = z - \text{score} \]

*Figure 1.29 Coefficient of Variation*

\[ CV = \frac{\sigma}{\bar{x}} = \frac{0.1028}{0.0538} = 1.9108 \]

*Figure 1.30 CV Calculation*

Transform to the z-score as in Figure 1.31:

\[ z - \text{score} = \frac{x_i - \bar{x}}{\sigma} \Rightarrow z - \text{score} = -\frac{\bar{x}}{\sigma} = -\frac{1}{CV} = -\frac{1}{1.9108} = -0.5233 \]

*Figure 1.31 Z-score Quantification*

Therefore, the area to the left of −0.5233 on a standard normal curve is the probability of loss for this investment as described by the composite-10, Case-Shiller index. Using the “= normsdist(z)” function in Microsoft Excel to calculate the value, we determine P(Loss) = 30.04%. Stated differently, there is a 69.94% chance that an investment in a project with the characteristics as quantified by the historic 15-year performance of the composite-10 will return the initial capital invested or more. Therefore, it is the efficiency measure that, when transformed utilizing a z-score and with the assumption of the underlying distribution being normal, provides the first and most important answer to the investment concern Return of Capital.
**Asset class consideration(s)**

In an example of asset class distributions, the three main asset classes are equity, fixed income, and real estate. To demonstrate an understanding of the differences, indices will be used to approximate each: equity (S&P 500), fixed income (VBMFX), and real estate (Case-Shiller composite-10). (See Figure 1.32.) For the period ending July 2010, the following return/risk characteristics were calculated on a historic 5- and 10-year basis.

What is striking is the difference when comparing the probability of loss for each asset class. Fixed income, as expected, has a significantly lower risk/return ratio. Equity and residential real estate have significantly higher probability of losses. For both periods of this analysis, the risk for equity is higher than real estate and the return, while higher in the 5-year case, may not warrant the risk of investment.

Although the returns can vary and the relationships between the asset classes adjust depending upon the historical period, it is important to view the investment, any investment, in any asset class as a distribution – i.e. with return and risk – rather than as simple return. Decisions made on asset investment without risk quantification fails to address the most simple investment premise, not losing money.

Further, an understanding of the return/risk dynamic is essential for an asset reposition. A slight change in either can drastically change asset performance and appearance. It should generally be a rule that when risk can be reduced at a greater rate than the return, the asset becomes more efficient.

A qualitative real estate example would be swapping out a single tenant in a building with another single tenant for the same lease terms. The difference is that the initial tenant could be a B-grade tenant while the new tenant is ExxonMobil, that is, AAA-rated. Because the lease terms have not changed, the expected return for the asset (building) is unchanged; however, the risk profile has significantly changed (i.e. risk has been reduced). This certainly would justify a change in tenant and may even justify a slight price-per-square-foot adjustment downward for ExxonMobil as acknowledgement of the higher-quality tenant. As described, this is a very basic, very successful asset reposition at the tenant level.

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<th>S&amp;P500</th>
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<th>Res RE</th>
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<td>Prob of Loss</td>
<td>48.61%</td>
<td>3.77%</td>
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<tr>
<td>Avg Return</td>
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<td>2.53%</td>
<td>37.41%</td>
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</tbody>
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* *Apr10’ - Dec01’

Figure 1.32 Asset Class Comparison
Pro forma portfolio modelling


