Medieval Modal Systems
Problems and Concepts

Paul Thom
MEDIEVAL MODAL SYSTEMS

This book explores noteworthy approaches to modal syllogistic adopted by medieval logicians including Abélard, Albert the Great, Avicenna, Averroes, Jean Buridan, Richard Campsall, Robert Kilwardby, and William of Ockham. The book situates these approaches in relation to Aristotle’s discussion in the Prior and Posterior Analytics, and other parts of the Organon, but also in relation to the thoughts of Alexander of Aphrodisias and Boethius on the one hand, and to modern interpretations of the modal syllogistic on the other. Problems explored include: Aristotle’s doctrine of modal conversion, the pure and mixed necessity-moods, modal ecthesis, the pure and mixed contingency-moods, and Aristotle’s use of counter-examples. Medieval logicians brought various concepts to bear on these problems, including the distinction between per se and per accidens terms, the notion of essential predication, the distinction between ut nunc and simpliciter propositions, the distinction between de dicto and de re modals, and the notion of ampliation. All these are examined in this book.
ASHGATE STUDIES IN MEDIEVAL PHILOSOPHY

Series Editors

John Marenbon, Trinity College, Cambridge, UK
Scott MacDonald, Cornell University, USA
Christopher J. Martin, University of Auckland, New Zealand
Simo Knuuttila, Academy of Finland and the University of Helsinki, Finland

The study of medieval philosophy is flourishing as never before. Historically precise and philosophically informed research is opening up this large but still relatively unknown part of philosophy's past, revealing—in many cases for the first time—the nature of medieval thinkers' arguments and the significance of their philosophical achievements. Ashgate Studies in Medieval Philosophy presents some of the best of this new work, both from established figures and younger scholars. Chronologically, the series stretches from c.600 to c.1500 and forward to the scholastic philosophers of sixteenth and early seventeenth-century Spain and Portugal. The series encompasses both the Western Latin tradition, and the Byzantine, Jewish and Islamic traditions. Authors all share a commitment both to historical accuracy and to careful analysis of arguments of a kind which makes them comprehensible to modern readers, especially those with philosophical interests.

Other titles in the series:

Theology at Paris, 1316–1345
Peter Auriol and the Problem of Divine Foreknowledge and Future Contingents
Chris Schabel
ISBN 0 7546 0204 4
Medieval Modal Systems

Problems and Concepts

PAUL THOM
Southern Cross University, Australia

Routledge
Taylor & Francis Group
LONDON AND NEW YORK
# Contents

*List of Figures*  
List of Tables  
Preface  
Acknowledgements  
List of Abbreviations  

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aristotle’s Modal Logic</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Late Antiquity</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>Abélard</td>
<td>43</td>
</tr>
<tr>
<td>4</td>
<td>Avicenna</td>
<td>65</td>
</tr>
<tr>
<td>5</td>
<td>Averröes</td>
<td>81</td>
</tr>
<tr>
<td>6</td>
<td>Kilwardby</td>
<td>93</td>
</tr>
<tr>
<td>7</td>
<td>Campsall</td>
<td>117</td>
</tr>
<tr>
<td>8</td>
<td>Ockham</td>
<td>141</td>
</tr>
<tr>
<td>9</td>
<td>Buridan</td>
<td>169</td>
</tr>
<tr>
<td>10</td>
<td>Conclusion</td>
<td>193</td>
</tr>
</tbody>
</table>

*Bibliography*  
*Index*  

vii
ix
xi
xiii
xiv

203
207
List of Figures

1.1 Non-modal square of opposition 8
2.1 Theophrastus's classification of necessities 22
2.2 Boethius's classification of necessities 39
3.1 Types of *de rebus* proposition 47
3.2 Modal square of opposition 52
3.3 Square of opposition for *de esse* modals 56
3.4 Square of opposition for *de non esse* modals 56
3.5 Abélard's *L/X/M* system and Aristotle's system 61
3.6 Abélard's *L/M* system and Aristotle's system 63
4.1 Avicenna's *L/M* system and Aristotle's system 71
4.2 Avicenna's *L/X/M* system and Aristotle's system 73
4.3 Relations among descriptionals 76
6.1 Kilwardby's broad-sense *L/X/M* * simpliciter* system and Aristotle's system 101
6.2 Kilwardby's 'natural' contingency *X/Q* * simpliciter* system and Aristotle's system 105
6.3 Kilwardby's 'natural' contingency *L/Q* system and Aristotle's system 106
6.4 Kilwardby's 'natural' contingency *L/X/Q* system and Aristotle's system 109
6.5 Kilwardby's 'indefinite' contingency *simpliciter* *X/Q/M* system and Aristotle's system 113
7.1 Campsall's classification of assertorics 121
7.2 Campsall's *L/M* system and Aristotle's system 124
7.3 Campsall's *L/X/M* system and Aristotle's system 128
7.4 Campsall's *L/X/M* *simpliciter* (or first-class *ut nunc*) system and Aristotle's system 131
7.5 Campsall's *X/Q* *simpliciter* (or first-class *ut nunc*) system and Aristotle's system 134
7.6 Campsall's *L/Q* system and Aristotle's system 135
7.7 Campsall's *L/X/Q* system and Aristotle's system 137
7.8 Campsall's *X/Q/M* system and Aristotle's system 138
8.1 Ockham's *simpliciter* *L/X/M* system and Aristotle's system 153
8.2 Ockham's *X/Q* system and Aristotle's system 158
8.3 Ockham's *L/Q* system and Aristotle's system 159
8.4 Ockham's *L/X/Q* system and Aristotle's system 160
8.5 Ockham's *L/Q/M* system and Aristotle's system 161
8.6 Ockham's *simpliciter* *X/Q/M* system and Aristotle's system 163
9.1 Modal octagon of opposition 171
9.2 Buridan’s L/M system and Aristotle’s system 174
9.3 Buridan’s L/X/M system and Aristotle’s system 179
9.4 Buridan’s *simpliciter* L/X/M system and Aristotle’s system 182
9.5 Buridan’s X/Q system and Aristotle’s system 184
9.6 Buridan’s *simpliciter* X/Q system and Aristotle’s system 185
9.7 Buridan’s L/Q system and Aristotle’s system 187
9.8 Buridan’s L/X/Q system and Aristotle’s system 188
9.9 Buridan’s *simpliciter* X/Q/M system and Aristotle’s system 189
10.1 Containments among L/M systems 193
10.2 Containments among L/X/M systems 194
10.3 Containments among X/Q systems 196
10.4 Containments among L/Q systems 196
10.5 Containments among L/X/Q systems 197
10.6 Containments among L/Q/M systems 197
10.7 Containments among X/Q/M systems 198
List of Tables

1.1 Modal equipollence 6
1.2 Mnemonic names for syllogisms 10
1.3 Aristotle’s system: L/M combinations 10
1.4 Aristotle’s system: L/X/M combinations 12
1.5 Aristotle’s system: Q/Q combinations 13
1.6 Aristotle’s system: X/Q combinations 13
1.7 Aristotle’s system: L/Q combinations 14
1.8 Aristotle’s system: L/X/Q combinations 14
1.9 Aristotle’s system: L/Q/M combinations 15
1.10 Aristotle’s system: X/Q/M combinations 16
3.1 *De sensu/de rebus*, compound/divided 46
3.2 *De esse* modals 55
3.3 *De non esse* modals 55
3.4 Abélard’s system: syllogistic L/X/M combinations 60
3.5 Abélard’s system: non-syllogistic L/M combinations 62
4.1 Avicenna’s system: L/M combinations 71
4.2 Correspondences between L/M and L/X/M moods 72
4.3 Avicenna’s system: L/X/M combinations 73
4.4 Descriptive syllogisms 79
5.1 Averroes’s system: *per se* L/M combinations 87
6.1 Kilwardby’s broad-sense system: L/M combinations 98
6.2 Kilwardby’s broad-sense *simpliciter* system: L/X/M combinations 101
6.3 Kilwardby’s system: ampliated Q/Q combinations 103
6.4 Kilwardby’s ‘natural’ contingency *simpliciter* system: X/Q combinations 105
6.5 Kilwardby’s system: ‘natural’ contingency L/Q combinations 106
6.6 Kilwardby’s ‘natural’ contingency system: L/X/Q combinations 110
6.7 Kilwardby’s ‘indefinite’ contingency system: L/Q/M combinations 111
6.8 Kilwardby’s ‘indefinite’ contingency *simpliciter* system: X/Q/M combinations 112
7.1 Campsall’s system: L/M combinations 124
7.2 Campsall’s system: L/X/M combinations 128
7.3 Campsall’s *simpliciter* (or first-class *ut nunc*) system: L/X/M combinations 130
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.4</td>
<td>Campsall's system: Q/Q combinations</td>
<td>132</td>
</tr>
<tr>
<td>7.5</td>
<td>Campsall's <em>simpliciter</em> (or first-class <em>ut nunc</em>) system: X/Q combinations</td>
<td>134</td>
</tr>
<tr>
<td>7.6</td>
<td>Campsall's system: L/Q combinations</td>
<td>135</td>
</tr>
<tr>
<td>7.7</td>
<td>Campsall's system: L/X/Q combinations</td>
<td>137</td>
</tr>
<tr>
<td>7.8</td>
<td>Campsall's system: X/Q/M combinations</td>
<td>137</td>
</tr>
<tr>
<td>8.1</td>
<td>Ockham's system: L/M combinations</td>
<td>148</td>
</tr>
<tr>
<td>8.2</td>
<td>Ockham's system: L/X/M combinations</td>
<td>151</td>
</tr>
<tr>
<td>8.3</td>
<td>Ockham's <em>simpliciter</em> system: L/X/M combinations</td>
<td>153</td>
</tr>
<tr>
<td>8.4</td>
<td>Ockham's Q/Q system with M- or Q-ampliation</td>
<td>156</td>
</tr>
<tr>
<td>8.5</td>
<td>Ockham's system: X/Q combinations</td>
<td>158</td>
</tr>
<tr>
<td>8.6</td>
<td>Ockham's system: L/Q combinations</td>
<td>159</td>
</tr>
<tr>
<td>8.7</td>
<td>Ockham's system: L/X/Q combinations</td>
<td>160</td>
</tr>
<tr>
<td>8.8</td>
<td>Ockham's system: L/Q/M combinations</td>
<td>161</td>
</tr>
<tr>
<td>8.9</td>
<td>Ockham's system: X/Q/M combinations</td>
<td>162</td>
</tr>
<tr>
<td>8.10</td>
<td>Ockham's <em>simpliciter</em> system: X/Q/M combinations</td>
<td>163</td>
</tr>
<tr>
<td>8.11</td>
<td>Ockham's system of compound modals: L/Q/M combinations</td>
<td>165</td>
</tr>
<tr>
<td>8.12</td>
<td>Ockham's system of compound modals with strong assertorics: L/X/Q/M combinations</td>
<td>166</td>
</tr>
<tr>
<td>9.1</td>
<td>Buridan's system: L/M combinations</td>
<td>173</td>
</tr>
<tr>
<td>9.2</td>
<td>Buridan's system: L/X/M combinations</td>
<td>179</td>
</tr>
<tr>
<td>9.3</td>
<td>Buridan's <em>simpliciter</em> system: L/X/M combinations</td>
<td>182</td>
</tr>
<tr>
<td>9.4</td>
<td>Buridan's system: Q/Q combinations</td>
<td>183</td>
</tr>
<tr>
<td>9.5</td>
<td>Buridan's system: X/Q combinations</td>
<td>184</td>
</tr>
<tr>
<td>9.6</td>
<td>Buridan's <em>simpliciter</em> system: X/Q combinations</td>
<td>185</td>
</tr>
<tr>
<td>9.7</td>
<td>Buridan's system: L/Q combinations</td>
<td>187</td>
</tr>
<tr>
<td>9.8</td>
<td>Buridan's system: L/X/Q combinations</td>
<td>188</td>
</tr>
<tr>
<td>9.9</td>
<td>Buridan's <em>simpliciter</em> system: X/Q/M combinations</td>
<td>189</td>
</tr>
<tr>
<td>9.10</td>
<td>Buridan's system of restricted modals: L/X/Q/M combinations</td>
<td>191</td>
</tr>
<tr>
<td>10.1</td>
<td>L/M systems</td>
<td>193</td>
</tr>
<tr>
<td>10.2</td>
<td>L/X/M systems</td>
<td>194</td>
</tr>
<tr>
<td>10.3</td>
<td>Q/Q systems</td>
<td>195</td>
</tr>
<tr>
<td>10.4</td>
<td>X/Q systems</td>
<td>196</td>
</tr>
<tr>
<td>10.5</td>
<td>L/Q systems</td>
<td>196</td>
</tr>
<tr>
<td>10.6</td>
<td>L/X/Q systems</td>
<td>197</td>
</tr>
<tr>
<td>10.7</td>
<td>L/Q/M systems</td>
<td>197</td>
</tr>
<tr>
<td>10.8</td>
<td>X/Q/M systems</td>
<td>198</td>
</tr>
<tr>
<td>10.9</td>
<td>Semantics for different systems</td>
<td>199</td>
</tr>
</tbody>
</table>
Preface

The major medieval systems of modal logic were developed during a period flanked by Pierre Abélard in the twelfth century and Jean Buridan in the fourteenth. Little of great significance pre-dates Abélard or post-dates Buridan; and these two Frenchmen are major figures indeed. In between their work lies that of the well-known Avicenna, Averroës and Ockham, as well as that of the comparatively unknown Robert Kilwardby in the thirteenth century and Richard Campsall in the early fourteenth. The work of these seven thinkers in the field of modal syllogistic will be my focus in this book.

These thinkers were motivated by two forces. First of all, there are the interpretive puzzles posed by Aristotle’s modal logic as expounded in the Prior Analytics and De Interpretatione. These two texts appear not always to be in accord with one another, and the modal syllogistic as presented in the Prior Analytics not only lacks a semantic foundation but appears to be internally inconsistent. All this is grist to the mill of the interpreting mind. Then there is the inherent fascination of modal logic as a field of theoretical enquiry. An uneasy relationship exists between the desire to interpret Aristotle and the desire to theorize modality. The latter, of course, is a philosophical desire, autonomous but also intimately connected with broader metaphysical and theological ways of thinking. The former force is, at first sight, a philological rather than a philosophical one. But in reality matters are not so simple, since the philosophical interpretation of any text – let alone one that increasingly carried such authority as that of Aristotle – is always partially governed by philosophical rather than philological imperatives.

There are two key theoretical questions stirring the minds of medieval modal logicians. The first question concerns the doctrine that I shall call actualism. According to this doctrine, modal propositions are about the actual things that ordinary non-modal propositions are about. Contrasted with actualism is the view that modal propositions are about what possibly falls under the subject-terms of the corresponding non-modal propositions. This doctrine I will call ampliationism. The leading actualists in the medieval period are Abélard, Ockham, and to a certain extent Campsall; the ampliationists are Avicenna and Buridan. A second question concerns the extent to which essentialist notions are assumed in the modal theories of our seven thinkers. All seven make use of the notion of an essential property. In addition, Averroës, Kilwardby and Campsall make use of the
notion of a kind, i.e. a class whose members necessarily share their essential properties.
Acknowledgements

Some parts of this book have seen earlier incarnations. Chapter One draws on the results of my book *The Logic of Essentialism: an interpretation of Aristotle's modal syllogistic*. Earlier versions of Chapter Three were delivered at the Philosophy Department of the University of Queensland, at the conference ‘Pierre Abélard, à l’aube des universités’, held at the University of Nantes and at a mini-conference on the History of Logic held at the University of Helsinki (2001). Earlier versions of Chapter Four were delivered at a History of Logic Conference held at the University of Cincinnati and at the Mulla Sadra Conference in Tehran (1999). An earlier version of Chapter Six was delivered at the Philosophy Department of Uppsala University (2001). An early version of Chapters Eight and Nine was delivered at the Ancient Philosophy Conference held at the Australian National University (1995). I would like to record my fond recollection of those learned discussions, and my gratitude to all who participated in them, especially Fred Johnson, Simo Knuuttila, Henrik Lagerlund, Constant Mews and Calvin Normore.

For her constant support and infectious good humour I thank Cassandra.
List of Abbreviations

L  Necessity
M  Possibility
Q  Contingency
X  Actuality
→  'is included in'
←  'includes'
|   'excludes'
⁻   'overlaps'
*  'necessarily' (governing a term)
†  'possibly' (governing a term)
‡  'contingently' (governing a term)
□  'it is necessary that'
◊  'it is possible that'
▽ 'it is contingent that'
~  Negation
Chapter 1

Aristotle’s Modal Logic

To begin at the beginning, we need to understand the essentials (so to speak) of Aristotle’s modal syllogistic, that being the foundation on which the whole of medieval modal logic is built. But no foundation is ultimate.

The Platonic background

Many ideas in Aristotle have their antecedents in Plato. In the case of the modal logic, we can find several such antecedents in Plato’s *Phaedo*, where Socrates is portrayed as traversing a three-stage intellectual journey. At first he held that a thing may be beautiful by virtue of the gold in it. Then he thought that a thing could be beautiful only by virtue of the beauty in it, and analogously that a thing could be hot only by reason of the heat in it. Finally, he recognized – in the case of heat – that something can be hot because of the fire in it (because fire is by its nature hot). A metaphysical logician reading this, may well have been set thinking about what is in effect a modal syllogism, concluding that heat inheres in a subject, on the ground that heat necessarily inheres in fire, and fire inheres in the subject.

Towards the end of the dialogue, Socrates advances his final argument in favour of the soul’s immortality, and this argument (so it happens) depends on a number of notions that will later resonate in modal syllogistic. Key among those notions are the following three.

1. The forms are supposed to exist and other things take their names from them;¹
2. Snow cannot be hot, while remaining snow, and similarly fire cannot be cold while remaining fire;²
3. While the opposition between heat and cold is, we could say, a primary or nomic one, there is a secondary opposition between fire and snow such that the snow retreats or else is annihilated by the advance of heat, and in general:

¹ Plato (1993) 102b.
² Plato (1993) 103d.
...it is not only the opposite that doesn’t admit its opposite; there is also that which brings up an opposite into whatever it enters itself; and that thing, the very thing that brings it up, never admits the quality opposed to the one that’s brought up.3

Each of these notions undergoes significant development at Aristotle’s hands, and each one of them will play a role in his modal syllogistic.

Aristotelian logic and metaphysics

The modal logicians of the Middle Ages saw their enterprise as linked with a motley assemblage of Aristotelian texts, drawn from the Categories, De Interpretatione, Prior and Posterior Analytics, the Topics and the Sophistical Refutations. The fundamental ideas in those texts are the idea of a term, a being, a categorical proposition and essential predication.

Terms

A term is a noun-phrase – any expression that could be subject or predicate in a categorical proposition. A term may or may not stand for a being, and accordingly even an expression like ‘non-being’, which explicitly excludes beings, is a term.4 Nonetheless, terms standing for beings constitute an important sub-class among terms. Beings, according to the Categories, fall into ten classes or categories. The first category comprizes the substances, and substance-terms notably include the names of natural kinds – terms like ‘man’, ‘horse’. Other categories are the qualities (including colours such as whiteness),5 and the relatives (including positions such as standing and sitting).6

Some terms do not directly designate anything that is in one of the categories. Some terms, for instance, are denominative, like ‘literate’ and ‘brave’ which, Aristotle states, are expressions deriving from ‘literacy’ and ‘bravery’ but with a different ending.7 Literacy (as a form of knowledge) and bravery (as a virtue) are beings in the category of quality;8 but the literate and the brave as such are not, even though the people who are

---

5 Aristotle (1963) ch.8, 9a28ff.
6 Aristotle (1963) ch.7, 6b11ff.
7 Aristotle (1963) ch.1, 1a12ff.
8 Aristotle (1963) ch.8, 8b29-35.
literate or brave are beings in the category of Substance. Denominative terms include ‘white’, ‘standing’ and ‘sitting’ (understood as adjectives), though the terms for the qualities or relatives from which these adjectives derive do stand for beings in the categories of Quality or Relative. Denominative terms, especially Aristotle’s paradigm ‘literate’, will figure in the arguments of the medieval modal logicians.

At first sight, Aristotle’s definition of denominative terms in the *Categories* seems hospitable to Platonism. It seems almost like a quote from Plato’s description of the way in which the name of a form gets applied to particulars that participate in the form. But there is a subversive element in Aristotle’s thinking here. Denominative terms may be semantically secondary to the abstract terms from which they derive, but according to Aristotle the primary realities are not the qualities and relatives denoted by those abstract terms but the substances underlying the denominative terms. On this point he is at odds with Plato who took the abstract terms from which denominative terms derive grammatically to stand for the primary realities.

Another group – compound terms – combine a number of terms, some of which may be denominative, for instance ‘musical Miccalus’, ‘intelligible Aristomenes’. Aristotle holds that in some cases the components in a compound term form a unity only accidentally. He gives the example of the terms ‘white’ and ‘musical’:

Of things predicated, and things they get predicated of, those which are said accidentally, either of the same thing or of one another, will not be one. For example, a man is white and musical, but ‘white’ and ‘musical’ are not one, because they are both accidental to the same thing.

The crucial thing here is whether the component terms are related to one another *per se* or *per accidens*.

Aristotle applies the *per selp per accidens* distinction to terms as well as to relations between terms. The distinction between *per se* and *per accidens* terms is explained in the *Posterior Analytics* as the third of four senses of ‘*per se*’ *[kath’ hauto]*. Something is *per se* in this sense when it:

... is not said of some other underlying subject – as what is walking is something different walking (and white), while a reality, and whatever signifies some this, is just what it is without being something else.

---

10 Aristotle (1963) 21a7-11.
Putting this together with the doctrine of the *Categories*, it seems that a *per se* term is a general term in the category of substance. However, there is also a more general sense in Aristotle according to which a general term in any category is *per se*.

One term may be related to another in a variety of interesting ways, four of which are theorized in Aristotle’s doctrine of the *predicables* – definition, genus (and differentia), peculiarity and accident. These constitute four types of relation between a predicate and its subject (assumed to be a *per se* term). The predicate may be either essential or non-essential (*per se* or *per accidens*) to that subject, and may independently be either convertible or non-convertible with it. This generates four possibilities according to whether the predicate is (1) convertible and essential (a definition), (2) non-convertible and essential (a genus or differentia), (3) convertible and non-essential (a peculiarity), or (4) non-convertible and non-essential (an accident).\(^{12}\)

According to this definition, an accident is what is predicated non-essentially and non-convertibly of the subject; according to a second definition, an accident is what can belong and can not belong to the subject.\(^{13}\) That the two definitions are not equivalent seems to be shown by the relation of the predicate ‘white’ to the subject ‘swan’: this predicate is not convertible with ‘swan’ and is not essential to ‘swan’, and yet it is not true that swans can be other than white (or at least Aristotle and his contemporaries thought so). The case is similar with the whiteness of snow (and indeed – to recall the *Phaedo* – with the coldness of snow or the hotness of fire). Which definition, then, is to be preferred? If we adopt the second definition, and if the classification of the four predicables is to be exhaustive, then there can be no such predicate as a necessary accident, because accidents are all *contingent* predicates of their subjects. On the contrary view, we can maintain the first definition of ‘accident’ whereby the essential is a proper subclass of the necessary, some accidents are contingent and some are necessary, and there is nothing to prevent one and the same predicate being a contingent predicate of some subjects and a necessary predicate of others.

**Propositions**

A categorical proposition is, according to Aristotle’s definition, ‘an expression which affirms or denies something of something’\(^ {14} \) – where that

---

\(^{12}\) Aristotle (1928) A8, 103b6-19.

\(^{13}\) Aristotle (1928) A5, 102b4ff.

which is affirmed or denied is the predicate and that of which it is affirmed or denied is the subject. The proposition is either affirmative or negative, and is either universal or particular, thus generating four propositional forms. In the Middle Ages the four forms were represented by the four vowels a (universal affirmative), e (universal negative), i (particular affirmative), and o (particular negative). Among the categorical propositions that Aristotle asserts or supposes to be true in the *Prior Analytics* are ‘Every animal is moving’, 15 ‘Every man is an animal’, 16 ‘Every animal is waking’, 17 ‘No animal is moving’, 18 ‘Nothing white is an animal’, 19 ‘No horse is good’, 20 ‘No horse is waking’, 21 ‘Nothing white is waking’, 22 ‘Something white is not an animal’, 23 and ‘Something white is not waking’. 24 In one passage Aristotle identifies a special class of assertorics, ones whose truth is not limited to any time but are true haplōs [simpliciter]. 25

Of special interest to us are modal propositions. These are categorical propositions to the effect that something belongs necessarily, or possibly, or contingently, to a subject. Contingency is understood as two-way possibility. We have to note that Aristotle distinguishes two ways of understanding contingency-propositions, either as ‘a is contingent for all that to which b belongs’ or ‘a is contingent for all that for which b is contingent’. 26 Following medieval usage, I will refer to the first of these senses as unampliated and the second as amplified. In the *Prior Analytics* Aristotle’s examples of true necessity-propositions include ‘Every man is necessarily an animal’, 27 ‘Every horse is necessarily an animal’, 28 ‘Something white is necessarily an animal’, 29 ‘Some man is necessarily an

15 Aristotle (1989) A9, 30a23-25, 28-33; 30b2-6; A11, 32a4-5.
18 Ibid.
animal', 30 ‘Some animal is necessarily biped’, 31 ‘Something white is necessarily not an animal’, 32 and ‘Some animal is necessarily not biped’. 33 His examples of true contingency-propositions include ‘Every man may or may not be white’, 34 ‘Every animal may or may not be white’, 35 ‘Something white may or may not be an animal’, 36 ‘Every horse may or may not be moving’, 37 ‘Every man may or may not be moving’, 38 ‘Every cloak may or may not be white’, 39 ‘Every animal may or may not be healthy’, 40 ‘Every horse may or may not be healthy’, 41 ‘Every man may or may not be healthy’, 42 and ‘Every man may or may not be sleeping’. 43

Table 1.1 Modal equipollence

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>‘possible to be’</td>
<td>‘not possible to be’</td>
<td>‘not impossible to be’</td>
<td>‘impossible to be’</td>
</tr>
<tr>
<td></td>
<td>‘not impossible to be’</td>
<td>‘possible not to be’</td>
<td>‘not necessary not to be’</td>
<td>‘necessary not to be’</td>
</tr>
<tr>
<td></td>
<td>‘not necessary not to be’</td>
<td>‘possible not to be’</td>
<td>‘not impossible not to be’</td>
<td>‘impossible not to be’</td>
</tr>
<tr>
<td></td>
<td>‘not necessary not to be’</td>
<td>‘possible not to be’</td>
<td>‘not impossible not to be’</td>
<td>‘necessary not to be’</td>
</tr>
</tbody>
</table>

In the De Interpretatione Aristotle recognizes that modal expressions are equipollent to one another as shown in Table 1.1. 44

---

44 The presentation follows that of Ackrill in Aristotle (1963) p.62. Ackrill ad 22a14 points out that this is Aristotle’s second attempt at diagramming these equipollences, and involves a correction of the earlier attempt.
Aristotle also includes 'contingent to be' and its internal and external negations in the table, and clearly intends 'contingent' to mean the same as 'possible', though in the Prior Analytics he differentiates contingency as two-sided possibility. In that text he distinguishes two types of contingency – that which is not necessary but happens most often and naturally (such as for humans to go grey), and that which happens or fails to happen by chance, such as for humans to walk.

The link between the two terms of a proposition may be either per se or per accidens. The Posterior Analytics explains that one term is predicated per se of another when it enters into the other’s definition or vice versa.

Sometimes it is unclear what proposition is to be understood as being expressed by a given sentence, because the sentence is ambiguous; and one type of ambiguity is especially relevant to modal logic – the ambiguity of scope that Aristotle called the fallacy of composition. In the Sophistical Refutations he discusses the sentences ‘It is possible for the sitting to walk’ and ‘It is possible for those not writing to write’. Each sentence contains a denominative word (which I have translated as ‘the sitting’ and ‘those not writing’), followed by an infinitive with an opposed sense. Aristotle points out that the sentences are ambiguous. On the one hand the modal expression may be inserted between the opposed words (‘For the sitting it is possible to walk’) dividing them from one another; and then the sense will be that the ability to walk or to write is attributed to someone who is not sitting or not writing. On the other hand, the opposed words may be compounded, and then the sense will be that what is possible is that those who are sitting are walking, or that those who are not writing are writing.

This Aristotelian analysis is fundamental to much of medieval modal logic.

Another important ambiguity is that unqualified [haplōs] statements of necessity differ from necessity-statements like 'Everything is necessarily is, so long as it is'.

The relations between universal propositions and particulars of the opposite quality is one of contradiction; that between two universals of opposite quality is contrariety. Two particulars of opposite quality are called sub-contraries. Universal propositions imply the corresponding particulars. The De Interpretatione contains Aristotle’s explications of the

47 Aristotle (1975) A4, 73b5ff.
48 Aristotle (1928) 166a22-30.
important relations of contrariety and contradiction. The universal affirmative and particular negative are contradictories, as are the universal negative and particular affirmative. The two universals are contraries. The fundamental logical relations (implication and contradiction) among categorical propositions that share a subject and predicate are shown in Figure 1.1. Conceptually, the relations of contrariety and subcontrariety are derivative on these. A contrary of $p$ is a proposition implying $p$’s contradictory, and a subcontrary of $p$ is a proposition implied by $p$’s contradictory.

From Table 1.1 we can infer the logical relation that holds among pairs of singular modal propositions. For instance we can see that ‘It is necessary that $x$ is $a$’ is contradictory to ‘It is possible that $x$ is not $a$’. What we cannot infer is the logical relations between two quantified modal propositions, such as ‘It is necessary that every $b$ is $a$’ and ‘It is possible that no $b$ is $a$’. There is indeed evidence that Aristotle clearly understood what these logical relations are in individual cases, but what is lacking in his discussion is a synoptic view of them, such as he presented for singular propositions. For such a synoptic view we must wait for Abélard.

![Figure 1.1 Non-modal square of opposition](image)

The *Prior Analytics* outlines the ways in which the subject and predicate of a categorical proposition can be interchanged while preserving truth.

---

51 Aristotle (1963) 17b3-6, 16-20.
This is the doctrine of conversion. Universal negative propositions ('No b is a') are convertible ('Therefore no a is b'), as are particular affirmatives ('Some b is a therefore some a is b'). Consequently universal affirmatives ('Every b is a') are partially convertible ('Therefore some a is b').

The conversion of necessity-propositions proceeds just as for non-modal propositions. The universal and particular affirmative both convert to a particular affirmative, and the universal negative to a universal negative. Contingency-propositions behave differently. Affirmatives and negatives are inter-changeable; and while universal or particular affirmatives convert to a particular affirmative, the universal negative does not convert. Bearing in mind that negatives are equivalent to affirmatives, a counter-example to the convertibility of the universal affirmative is also a counter-example to the convertibility of the universal negative. Aristotle supplies such an example, taking it as true that every man may or may not be white (i.e. pale), but false that everything that is pale may or may not be a man.

*Syllogisms*

If two categorical propositions sharing a (middle) term are combined with each other, they fall into one of three *figures* depending on whether the middle term is (1) subject in one proposition and predicate in the other, or (2) predicate in both, or (3) subject in both. In each figure, there are some combinations of premises that imply a conclusion containing the terms in the premises other than the middle. These are the *syllogisms*. Leaving aside derivative cases where a premise is strengthened or a conclusion weakened, the fundamental syllogisms are listed in Table 1.2.

One syllogism can be equivalent to another, in the sense that it is valid if and only if the other is valid. Aristotle uses two techniques for demonstrating these equivalences – direct and indirect reduction. Both types of reduction involve the transformation of one syllogism into another by means of applying a rule that preserves validity. In the case of direct reduction the transformation involves replacing a premise or conclusion by its equivalent converse. In the case of indirect reduction the transformation is more complex: one premise is left constant, the contradictory of the other

---

57 For this expression see Thom (1981) §29.
premise becomes the conclusion, and the contradictory of the conclusion becomes a premise.\textsuperscript{58}

Table 1.2 Mnemonic names for syllogisms

<table>
<thead>
<tr>
<th>Figure</th>
<th>Mood</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>aaa</td>
<td>Barbara</td>
</tr>
<tr>
<td></td>
<td>eae</td>
<td>Celarent</td>
</tr>
<tr>
<td></td>
<td>aii</td>
<td>Darii</td>
</tr>
<tr>
<td></td>
<td>eio</td>
<td>Ferio</td>
</tr>
<tr>
<td>2</td>
<td>eae</td>
<td>Cesare</td>
</tr>
<tr>
<td></td>
<td>aea</td>
<td>Camestres</td>
</tr>
<tr>
<td></td>
<td>eio</td>
<td>Festino</td>
</tr>
<tr>
<td></td>
<td>aoa</td>
<td>Baroco</td>
</tr>
<tr>
<td>3</td>
<td>aii</td>
<td>Datisi</td>
</tr>
<tr>
<td></td>
<td>iai</td>
<td>Disamis</td>
</tr>
<tr>
<td></td>
<td>eio</td>
<td>Ferison</td>
</tr>
<tr>
<td></td>
<td>oao</td>
<td>Bocardo</td>
</tr>
</tbody>
</table>

Table 1.3 Aristotle's system: L/M combinations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LLL,\textsuperscript{59} [LMM],\textsuperscript{60} [MLM]\textsuperscript{61}</td>
</tr>
<tr>
<td>2</td>
<td>LLL, [LMM],\textsuperscript{62} [MLM]\textsuperscript{63}</td>
</tr>
<tr>
<td>3</td>
<td>LLL, [LMM],\textsuperscript{64} [MLM]\textsuperscript{65}</td>
</tr>
</tbody>
</table>

\textsuperscript{58} Aristotle seems to take the procedure of direct reduction as obvious. He states the principle underlying indirect reduction at Aristotle (1989) B8, 51b1-5.

\textsuperscript{59} All the LLL moods are asserted at Aristotle (1989) A8, 29b36-30a2.

\textsuperscript{60} These are equivalent by indirect reduction to the second-figure LLL moods. Aristotle does not assert these moods, however at Aristotle (1989) A16, 35b38-36a2, 15-17, 39-b2 he does assert the first-figure LQM moods.

\textsuperscript{61} These are equivalent by indirect reduction to the third-figure LLL moods. Aristotle does not assert these moods, however at Aristotle (1989) A16, 36a39-b2 he does assert Ferio QLM.

\textsuperscript{62} These are equivalent by indirect reduction to the first-figure LLL moods. The LMM moods are not asserted by Aristotle, but at Aristotle (1989) A19, 38a16-18, b25-27 he does assert Cesare and Festino LQM.

\textsuperscript{63} These are equivalent by indirect reduction to the third-figure LLL moods. They are not asserted by Aristotle, however at Aristotle (1989) A19, 38a21-26 he does assert Camestres QLM.
For purposes of abbreviation, I shall use the letters ‘X’, ‘L’, ‘M’ and ‘Q’ respectively to indicate assertoric propositions, necessity-propositions, possibility- and contingency-propositions. Aristotle’s modal syllogisms fall into eight layers, distinguished by the combinations of modalities that occur in them. The first layer is characterized by the presence of necessity- and/or possibility-propositions. Aristotle accepts the syllogisms displayed in Table 1.3. He takes the first-figure LLL moods to be perfect. The LLL moods in the other figures (apart from Baroco and Bocardo) he reduces to the first figure by means of necessity-conversion. Baroco LLL is proved by *ecthesis*. If some $c$ cannot be $a$, then we may take some term ‘$d$’ such that every $d$ must be $c$ and no $d$ can be $a$; if then we also assume that every $b$ must be $a$, we have the premises of a Camestres LLL: ‘Every $b$ must be $a$, and no $d$ can be $a’. And from these premises we can infer that no $d$ can be $b$. But since every $c$ must be $d$, we can further infer that some $c$ cannot be $b$. A similar proof is available for Bocardo LLL. The remaining moods in Table 1.3 are reducible to LLL syllogisms by indirect reduction. I have bracketed the moods that Aristotle does not formulate explicitly.

A second layer of syllogisms combines assertoric propositions with necessity- and/or possibility-propositions. Aristotle accepts the combinations listed in Table 1.4. He takes the LXL moods in Figure 1 to be perfect. The LXL and XLL moods in the other figures he reduces to the first figure by means of modal or non-modal conversion. The remaining moods are equivalent to one or other of these by indirect reduction.

Some of the absences from this table are noteworthy. First, while Aristotle accepts Barbara LXL he rejects Barbara XLL. Supposing that every animal is moving, and given that every man is necessarily an animal, Barbara XLL would imply the falsehood that every man is necessarily moving. This question of the two Barbaras will be a thorn in the side of many a post-Aristotelian logician. Second, Aristotle thinks Baroco XLL and Bocardo LXL are invalid and proposes counter-examples to them. In the case of Baroco XLL his reasoning as it survives in our texts of the *Prior Analytics* is sketchy. he says that a counter-example can be constructed

---

64 These are equivalent by indirect reduction to the second-figure LLL moods. Aristotle does not assert these moods, however at Aristotle (1989) A22, 40a39-b1 he does assert Datisi LQM.
65 These are equivalent by indirect reduction to the first-figure LLL moods. The MLM moods are not asserted by Aristotle, however at Aristotle (1989) A22, 40a39-b3 he does assert Disamis and Bocardo QLM.
using ‘the same terms’, and if we track that reference back we find the
terms ‘animal’, ‘man’, ‘white’.

It should be mentioned that the syllogisms in Table 1.3 can be seen as
derivative on those in Table 1.4, strengthening assertoric premises into
necessity-premises – though Aristotle did not see them in this way.

Table 1.4 Aristotle’s system: L/X/M combinations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LXL(^{68})</td>
</tr>
<tr>
<td></td>
<td>[Celarent Ferio LMX(^{69}); Darii XMM;(^{70})</td>
</tr>
<tr>
<td></td>
<td>Celarent MLX;(^{71}) Darii Ferio MXM(^{72})]</td>
</tr>
<tr>
<td>2</td>
<td>Cesare Festino LXL;(^{73}) Camestres XLL</td>
</tr>
<tr>
<td></td>
<td>[LMX;(^{74}) Cesare Camestres MLX;(^{75}) Festino MXM(^{76})]</td>
</tr>
<tr>
<td>3</td>
<td>Datisi Ferison LXL, Disamis XLL(^{77})</td>
</tr>
<tr>
<td></td>
<td>[Ferison LMX;(^{78}) MXM;(^{79}) Datisi Disamis XMM(^{80})]</td>
</tr>
</tbody>
</table>

\(^{68}\) The first-figure LXL moods are asserted at Aristotle (1989) A9, 30a17-23, 37-b1.

\(^{69}\) These are equivalent by indirect reduction to Festino and Cesare LXL. These moods are not asserted by Aristotle, but at Aristotle (1989) A16, 36a8-15, 34-39 he does assert Celarent and Ferio LQX.

\(^{70}\) This is equivalent by indirect reduction to Camestres XLL. This mood is not asserted by Aristotle, however Darii XQM is provable from Ferison LQX. See Thom (1996) p.71.

\(^{71}\) This is equivalent by indirect reduction to Disamis XLL. This mood is not asserted by Aristotle.

\(^{72}\) These are equivalent by indirect reduction to Ferison and Datisi LXL. These moods are not asserted by Aristotle.

\(^{73}\) The second-figure moods are asserted at Aristotle (1989) A10, 30b9-18 and 31a5-10.

\(^{74}\) These are equivalent by indirect reduction to the first-figure LXL moods. These moods are not asserted by Aristotle, however at Aristotle (1989) A19, 38a16-18, b25-27 he does assert Cesare and Festino LQX.

\(^{75}\) These are equivalent by indirect reduction to Datisi and Ferison LXL. These moods are not asserted by Aristotle, but Camestres QLX is asserted at Aristotle (1989) A19, 38a21-26.

\(^{76}\) This is equivalent by indirect reduction to Disamis XLL. This mood is not asserted by Aristotle.

\(^{77}\) The third-figure moods are asserted at Aristotle (1989) A11, 31b12-20 and 35-37.
The third layer of modal syllogisms consists entirely of contingency-propositions. Aristotle's system is very simple and is shown in Table 1.5. Once again Aristotle takes the first-figure syllogisms to be perfect, the remaining ones reducing to them by means of conversion. Absent from the table are all second-figure moods. This is because of the failure of contingency-conversion for universal negative contingency-propositions. Also noteworthy is the presence of Bocardo QQQ, which, because of complementary conversion is equivalent to Disamis QQQ. (Similarly Darii QQQ is equivalent to Ferio QQQ, and so forth.)

Table 1.5 Aristotle's system: Q/Q combinations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>QQQ(^{81})</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>QQQ(^{82})</td>
</tr>
</tbody>
</table>

The fourth layer combines contingency-propositions with assertorics. Again, the first-figure syllogisms are taken to be perfect, and the others reduce by conversion. Again, there are no second-figure moods, for the same reason as with the uniform contingency-moods. These results are displayed in Table 1.6.

Table 1.6 Aristotle's system: X/Q combinations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>QXQ(^{83})</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Disamis XQQ; Datisi (Ferison) QXQ(^{84})</td>
</tr>
</tbody>
</table>

\(^{78}\) This is equivalent by indirect reduction to Camestres XLL. This mood is not asserted by Aristotle, but Ferison LQX is asserted at Aristotle (1989) A22, 40b3-6.

\(^{79}\) These are equivalent by indirect reduction to the first-figure LXL moods. These moods are not asserted by Aristotle, however at Aristotle (1989) A21, 39b31-39 he does assert Bocardo QXM.

\(^{80}\) These are equivalent by indirect reduction to Cesare and Festino LXL. These moods are not asserted by Aristotle, however Datisi and Disamis XQM are provable from Bocardo QXM. See Thom (1996) p.76.

\(^{81}\) The QQQ first-figure moods are asserted at Aristotle (1989) A14, 32b38-a5, 33a21-27.

\(^{82}\) The third-figure moods are asserted at Aristotle (1989) A20, 39a14-23, 31-38.

\(^{83}\) The first-figure moods are asserted at Aristotle (1989) A15, 33b33-40, 35a30-35.
The fifth layer combines contingency- and necessity-propositions. The results are displayed in Table 1.7, where again the first-figure moods are said to be perfect, the others reducing to them by means of conversion. The syllogisms in this layer might be seen as derivative on those in Table 1.6, strengthening assertoric premises into necessity-premises — though Aristotle did not see them in this way.

**Table 1.7 Aristotle's system: L/Q combinations**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>QLQ&lt;sup&gt;85&lt;/sup&gt;</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Disamis LQQ; Datisi (Ferison) QLQ&lt;sup&gt;86&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

The sixth layer of syllogisms draws assertoric conclusions from necessity- and contingency-premises. These are shown in Table 1.8. None are perfect. The LQX and QLX moods (apart from Festino LQX) are reduced by Aristotle indirectly to first-figure LXL moods. Festino LQX reduces by modal conversion to Ferio LQX.

**Table 1.8 Aristotle's system: L/X/Q combinations**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Celarent Ferio LQX&lt;sup&gt;87&lt;/sup&gt;</td>
</tr>
<tr>
<td>2</td>
<td>LQX, QLX&lt;sup&gt;88&lt;/sup&gt;</td>
</tr>
<tr>
<td>3</td>
<td>Ferison LQX&lt;sup&gt;90&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>84</sup> The third-figure moods are asserted at Aristotle (1989) A21, 39b26-30.
<sup>85</sup> The first-figure QLQ moods are asserted at Aristotle (1989) A16, 35b23-26.
<sup>86</sup> The third-figure QLQ moods are asserted at Aristotle (1989) A22, 40a16-23, 39-b3.
<sup>89</sup> Camestres is proved at Aristotle (1989) A19, 38a21-26. Cesare QLX is rejected at Aristotle (1989) A19, 38a38-b5 but can be proved by Aristotelian methods from Darii LXL or Darii QXQ: see Thom (1996) p.129. Baroco QLX can be proved by Aristotelian methods from Barbara QXM: see Thom (1996) p.63. Festino QLX can be proved analogously from Celarent QXQ.
<sup>90</sup> Ferison is proved from Ferio LQX at Aristotle (1989) A22, 40b3-6.
The seventh layer of syllogisms contains a combination of modalities—necessity-, possibility- and/or contingency-propositions, but always with a possibility-conclusion. These are shown in Table 1.9.

There are no perfect moods here. Aristotle demonstrates the validity of Barbara and Darii LQM by indirect reduction to second-figure LLL moods. He demonstrates the validity of Ferio QLM by indirect reduction to Datisi LLL. Second- and third-figure LQM and QLM moods are reduced to the first figure by modal conversion with the exception of Bocardo QLM, which is reduced indirectly to Barbara LLL. Indirect reduction could have been used more widely, to reduce third-figure LQM moods to first-figure LQM moods, and second-figure QLM moods to first-figure QLM moods. All the syllogisms in Table 1.9 could be seen as derivative on those in Table 1.3, strengthening possibility-premises into contingency-premises, the LQM and QLM syllogisms here being consequential upon the LMM and MLM syllogisms there.

**Table 1.9 Aristotle’s system: L/Q/M combinations**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LQM, QLM</td>
</tr>
<tr>
<td>2</td>
<td>LQM, QLM</td>
</tr>
<tr>
<td>3</td>
<td>LQM, QLM</td>
</tr>
</tbody>
</table>

The eighth and final layer draws possibility-conclusions from assertoric and contingency-premises. Aristotle makes it clear that the assertoric premises in these syllogisms are simpliciter [haplōs]. The procedure whereby he claims to demonstrate the validity of Barbara XQM (‘Every b is a, every c may or may not be b, so it is possible that every c is a’) is puzzling. He takes the proposition ‘It is necessary that some c is not a’, which is incompatible with the conclusion, and adds the assertoric corresponding to the contingency-minor (‘every c is b’), and (using a form of Bocardo) infers that some b is not a—which is incompatible with the

---

91 The first-figure LQM moods are asserted at Aristotle (1989) A16, 35b38-36a2, 15-17, 39-b2.
93 The second-figure LQM and QLM moods are stated at Aristotle (1989) A19, 38a16-18, 21-26; 38b25-27.
original major premise that \( a \) belongs to all \( b \). The puzzlement caused by this procedure can be crystallized into two questions. Why is Aristotle entitled to take the assertoric corresponding to the contingency-minor, rather than the contingency-minor itself? And, what exactly is the Bocardo to which Aristotle thinks he is reducing this Barbara?

Table 1.10 Aristotle's system: X/Q/M combinations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>XQM; 97 QXM 98</td>
</tr>
<tr>
<td>2</td>
<td>XQM; 99 QXM 100</td>
</tr>
<tr>
<td>3</td>
<td>XQM; 101 QXM 102</td>
</tr>
</tbody>
</table>

The syllogisms mentioned in Tables 1.9 and 1.10 could be seen as derivative on those in Table 1.4, strengthening possibility-premises into contingency-premises, the LQX, QLX, QXM, XQM syllogisms here being consequential upon the LMX, MLX, MXM and XMM syllogisms there. Aristotle does not adopt this perspective, but the thought prompts the observation that, since there are some combinations in Tables 1.9 and 1.10 that lack counterparts in Table 1.4, perhaps those absent counterparts would have been accepted by Aristotle. I list the moods in question:

**Conjecture 1.1.** Barbara, Celarent and Ferio XMM, Barbara MXM, Cesare and Festino XMM, Baroco MLX, Datisi Disamis and Bocardo LMX, and Datisi and Disamis MLX, are valid.

---

97 The first-figure XQM moods are asserted at Aristotle (1989) A15, 34a34-b1, 19-27; 35a30-31, 35-b2.
98 These moods could be seen as derivative on the first-figure QXQ moods. They can also be reduced, by Aristotle's procedure, to third-figure moods.
99 Cesare and Festino XQM are proved at Aristotle (1989) A18, 37b24-29 and 38a3-4. All the second-figure XQM moods can however be reduced, by Aristotle's procedure, to third-figure moods, Camestres to Ferison and Baroco to Bocardo.
100 Camestres QXM is proved at Aristotle (1989) A18, 37b24-29. The second-figure moods can be reduced, by Aristotle's procedure, to the first figure.
101 Datisi and Ferison XQM are proved at Aristotle (1989) A21, 39b26-31. All the third-figure moods can be reduced, by Aristotle’s procedure, to first-figure moods, Disamis to Celarent and Bocardo to Barbara.
102 Disamis QXM is proved at Aristotle (1989) A21, 39b26-31. The third-figure moods can be reduced, by Aristotle’s procedure, to the second figure.
Formal analysis

It will be useful to have a uniform and rigorous representation of the various logical theories to be discussed in this book, and to achieve this end I am going to use a formal notation, which I will now explain. I will represent general terms by lower-case letters such as ‘a’, ‘b’, ‘c’, ... I do not presuppose that these terms have existential import; in other words, if the letter ‘w’ stands for what is white, I do not imply by the use of this letter that there actually exist any white things. To indicate existential import I will underline the term-letter, so that ‘\(a\)' will indicate that something is a. I shall use an arrow to indicate being-included-in, so that ‘\(b \rightarrow a\)' will mean that the bs are included in the as. Equivalently, ‘\(a \leftarrow b\)' will mean that the as include the bs. The backwards arrow then signifies inclusion. I shall use a vertical stroke to signify exclusion, so that ‘\(b \mid a\)' means that the bs exclude the as. And I shall use the sign ‘\(\_
\)' to indicate overlap. With this notation we are able to state truth-conditions for non-modal categorical propositions as in definitions 1.1 to 1.4.

**Definition 1.1 (a).** ‘Every b is a’: \(b \rightarrow a\).
**Definition 1.2 (e).** ‘No b is a’: \(b \mid a\).
**Definition 1.3 (i).** ‘Some b is a’: for some ‘\(d\)’, \(b \leftarrow d \rightarrow a\).
**Definition 1.4 (o).** ‘Some b is not a’: for some ‘\(d\)’, \(b \leftarrow d \mid a\).

We also need to explicate the logical relation between ‘\(-\)’ and ‘\(\mid\)’:

**Definition 1.5 (\(\_\)).** ‘\(b \_ a\)’ is true iff ‘\(b \mid a\)’ is not true.

Note that in definition 1.1, ‘\(b\)’ is required to have existential import, whereas this is not so in definition 1.2 because negative propositions do not have existential import. In Definition 1.3 we need to stipulate that ‘\(d\)’ is non-empty, otherwise there is no guarantee that some b is a. However, in definition 1.4 there is no need to do this. ‘Some unicorn is not a saint’ is true because ‘unicorn’ is included in ‘unicorn’ and excludes ‘saint’. Clearly we may take any formula of the following types as axiomatic.

**Axiom 1.1.** If \(b \rightarrow a\) then \(b \_ a\).
**Axiom 1.2.** If \(b \mid a\) then \(a \mid b\).
**Axiom 1.3.** If \(c \rightarrow b \rightarrow a\) then \(c \rightarrow a\).
**Axiom 1.4.** If \(c \rightarrow b \mid a\) then \(c \mid a\).
**Axiom 1.5.** If \(b \rightarrow a\) then for some ‘\(d\)’, \(b \leftarrow d \mid a\).
**Axiom 1.6.** If \(b \_ a\) then for some ‘\(d\)’, \(b \leftarrow d \rightarrow a\).

These last two axioms, in order to be plausible, will need to read as asserting the constructibility rather than the existence in a particular language of a term ‘\(d\)’ under the stated conditions. Aristotle showed his awareness of this difference when he stipulated that the word ‘cloak’ stand
for ‘white man’. Among the theorems we can deduce from these axioms, the following are worth noting.

**Theorem 1.1.** If for some ‘d’, \( d \rightarrow b \) and \( d \mid a \) then not \( (b \rightarrow a) \). Suppose \( b \rightarrow a \); then since \( d \rightarrow b \), we have the chain ‘\( d \rightarrow b \rightarrow a' \)’, which simplifies to ‘\( d \rightarrow a' \)’ by Axiom 1.3, and this implies ‘\( d \rightarrow a' \)’ by Axiom 1.1, which by Definition 1.5 is inconsistent with our premise ‘\( d \mid a' \)’.

**Theorem 1.2.** If for some ‘d’, \( d \rightarrow b \) and \( d \rightarrow a \) then \( b \wedge a \). Suppose ‘\( b \wedge a \)’ is not true. Then by Definition 1.5, \( b \mid a' \); so since \( d \rightarrow a \), we have the chain ‘\( d \mid b' \)’, which simplifies to ‘\( d \mid b' \)’ by Axiom 1.4, and this is inconsistent with our premise ‘\( d \rightarrow b' \)’ by Axiom 1.1 and Definition 1.5.

Theorems 1.1 and 1.2 in conjunction with Axioms 1.5 and 1.6, entitle us to express any particular (as opposed to universal) proposition in either of two ways – as the denial of an inclusion or exclusion, or else in terms of the existence of a term ‘d’ linked by inclusion and/or exclusion to the particular proposition’s subject and predicate.

Sometimes it will be useful to have a way of representing what necessarily, or possibly, or contingently, falls under a given term. To do this I will superscribe the symbols ‘\( * \)’, ‘\( \dagger \)’, and ‘\( \ddagger \)’, respectively to the term-letter. Thus if ‘\( w \)’ stands for what is white, ‘\( w^* \)’ will stand for what is necessarily white, ‘\( w^\dagger \)’ for what is possibly white, and ‘\( w^{\ddagger} \)’ for what is contingently white (prescinding from the question whether anything is necessarily white etc.). The following axioms govern these notions.

**Axiom 1.7.** \( a^* \rightarrow a \).

**Axiom 1.8.** \( a \rightarrow a^\dagger \).

**Axiom 1.9.** \( a^\dagger \rightarrow a^* \).

**Axiom 1.10.** \( a^\ddagger \mid a^* \).

The proposition stating that it is necessary that \( p \) I will represent as ‘\( \Box p \)’.

The following axioms and rules govern this notion.

**Axiom 1.11.** If \( \Box p \) then \( p \).

**Axiom 1.12.** If \( \Box b \rightarrow a \) then \( \Box b^* \rightarrow a^* \).

**Axiom 1.13.** If \( \Box b \mid a \) then \( \Box b^\dagger \mid a^\dagger \).

**Axiom 1.14.** If \( \Box b \rightarrow a \) then \( \Box b^\dagger \rightarrow a^\dagger \).

**Axiom 1.15.** If \( \Box b \mid a \) then \( \Box b^{\ddagger} \mid a^{\ddagger} \).

**Axiom 1.16.** If \( \Box b^{\ddagger} \mid a \) then \( \Box b^\ddagger \mid a^* \).

---


105 An analogue of Axioms 1.12 and 1.13 in propositional modal logic is \( CLCpqLCLpLq \), which is a thesis of system T – Hughes and Cresswell (1968) ch.7, A6. An analogue of Axioms 1.14 and 1.15 is \( CLCpqLCPqMPq \), which is also
Rule 1.1. If ‘p’ is a thesis then ‘◇ p’ is a thesis.
Rule 1.2. If ‘If p then q’ is a thesis then ‘If (◇ p) then (◇ q)’ is a thesis.
Rule 1.3. If ‘If p and q then r’ is a thesis then ‘If (◇ p) and (◇ q) then (◇ r)’ is a thesis.

With these notational tools, it will be possible to represent most of the kinds of propositions we will encounter in our survey of medieval modal logic. Moreover, we can string several propositions together in this notation, so that a notation like ‘a → b → b* | c*’ will mean that whatever is a is b, and whatever is b is necessarily b, and nothing that is necessarily b is necessarily c. Such chains can be transformed in three ways — by shortening, lengthening, or by replacing one link by another. We are entitled to shorten a chain ‘c-b-a’ to ‘c-a’ when there exists a thesis that if c-b and b-a then c-a; this process I call simplification. We are entitled to lengthen a chain ‘c-b’ to ‘c-b-a’ when there exists a thesis that b-a; this process I call interpolation. We are entitled to substitute ‘d-c’ for ‘b-a’ when there exists a thesis that if the former then the latter; this process I call replacement.

Essentialist notions

We can define a term ‘a’ as being per se provided that it is necessary that whatever is a is necessarily a.
Definition 1.6. ‘a’ is per se iff □ a → a*. ‘Horse’ is a per se term according to Aristotle. ‘White’ is not a per se term, because, even if it happened that all white things were necessarily white, it would still be possible that something white was not necessarily white. To signal that a term is per se I shall write it in uppercase. Thus ‘C → a’ means that the cs are among the as and ‘c’ is per se.
Definition 1.7. ‘a’ is per accidens iff □ a → a^2. ‘Walking’ is a per accidens term. In addition to these types, for the sake of completeness, we need to add a third type, which we may call mixed terms ‘a’, whose defining characteristic is that some as are necessarily a and some are not, e.g., ‘white’. (Thus Aristotle’s two examples of terms that are not per se are of these two distinct types.)
Definition 1.8. ‘a’ is mixed iff it is neither per se nor per accidens. Medieval authors paid scant attention to what I am calling mixed terms. Elsewhere I have called these Quasi-Kind terms.\textsuperscript{106}

\textsuperscript{106} Thom (1996) p.316.
Besides all this purely logical apparatus, we will need on occasion to make use of some postulates that appear to be assumed in Aristotle’s metaphysics. I state three.\textsuperscript{107}

**Postulate 1.1.** If $x \rightarrow a^*$ then for some ‘$d$’, $x \rightarrow D$ and $\square d \rightarrow a$. This postulate does not state a formal logical principle, but it seems that Aristotle’s essentialism assumes it. What it ensures that whatever is essentially $a$ belongs to some kind that is necessarily included under ‘$a$’. For example anything that is essentially white must belong to a kind that is necessarily included among white things (swans, for example, or snow).

**Postulate 1.2.** If $x \not\vdash a^t$ then for some ‘$d$’, $x \rightarrow D$ and $\square d \not\vdash a^t$.

**Postulate 1.3.** If $x \rightarrow a^*$ and $\square B \rightarrow a^*$ and $x \leftrightarrow B$ then for some ‘$c$’, $\square C \rightarrow a$ and $x \rightarrow C$ and $\square C \models B$. These again are not formal logical principles, but seem to be assumed by two of Aristotle’s stated principles: that two intersecting species must both be species of some higher genus,\textsuperscript{108} and that whatever is in a genus is in some immediate species of that genus.\textsuperscript{109}

---


\textsuperscript{108} Aristotle (1928) \Delta2, 121b24-122a2.

\textsuperscript{109} Aristotle (1928) \Delta2, 122a26-27.
References


Aristotle (1963), Categories and De Interpretation, translated with notes by John Ackrill, Clarendon, Oxford.


Buridan, J. (1499), Perutile compendium totius logice Ioannis Buridani cum preclarissima solertissimi viri Joannis Dorp expositione, Venice; reprinted Minerva, Frankfurt.


Ockham (1962), Summa Logicae Pars Secunda et Tertiae Prima, edited by Philotheus Boehner, The Franciscan Institute, St. Bonaventura NY.


