“This relevant and timely book situates research and practice in the field of early childhood education. Its international context demonstrates the power of collaborative study and writing. The mathematical content is structured by using Bishop’s six universal activities. Connecting these mathematical ideas with classroom activities offers the reader a novel perspective on both early childhood research and practice.”

— Alan J. Bishop, Emeritus Professor, Monash University, Australia
MATHEMATICS IN EARLY CHILDHOOD

Structured around Bishop’s six fundamental mathematical activities, this book brings together examples of mathematics education from a range of countries to help readers broaden their view on maths and its interrelationship to other aspects of life.

Considering different educational traditions and diverse contexts, and illustrating theory through the use of real-life vignettes throughout, this book encourages readers to review, reflect on, and critique their own practice when conducting activities on explaining, counting, measuring, locating, designing, and playing.

Aimed at early childhood educators and practitioners looking to improve the mathematics learning experience for all their students, this practical and accessible guide provides the knowledge and tools to help every child.

Oliver Thiel is Associate Professor at Queen Maud University College for Early Childhood Education, Norway.

Elena Severina is Associate Professor at Western Norway University of Applied Sciences, Norway.

Bob Perry is Emeritus Professor, Charles Sturt University, Australia.
TOWARDS AN ETHICAL PRAXIS IN EARLY CHILDHOOD

Written in association with the European Early Childhood Education Research Association (EECERA), titles in this series will reflect the latest developments and most current research and practice in early childhood education on a global level. Feeding into and supporting the further development of the discipline as an exciting and urgent field of research and high academic endeavour, the series carries a particular focus on knowledge and reflection, which has huge relevance and topicality for those at the front line of decision making and professional practice.

Rather than following a linear approach of research to practice, this series offers a unique fusion of research, theoretical, conceptual and philosophical perspectives, values and ethics, and professional practice, which has been termed “Ethical Praxis.”

Other titles published in association with the EECERA:

**Listening to Children’s Advice about Starting School and School Age Care**
*Edited by Sue Dockett, Jóhanna Einarsdóttir, and Bob Perry*

**Understanding Pedagogic Documentation in Early Childhood Education**
*Revealing and Reflecting on High Quality Learning and Teaching*
*Edited by João Formosinho and Jan Peeters*

**A Vygotskian Analysis of Children’s Play Behaviours**
*Beyond the Home Corner*
*Edited by Zenna Kingdon*

**Mathematics in Early Childhood**
*Research, Reflexive Practice and Innovative Pedagogy*
*Edited by Oliver Thiel, Elena Severina, and Bob Perry*

**Exploring Career Trajectories of Men in the Early Childhood Education and Care Workforce**
*Edited by David L. Brody, Kari Emilsen, Tim Rohrmann, and Jo Warin*

For more information about this series, please visit: www.routledge.com/education/series/EECERA
CONTENTS

Contributors xiii
Foreword xx

Tony Bertram and Chris Pascal

1 Reflexivity and early childhood mathematics education: applying Bishop’s universality to vignettes of young children’s learning 1

Bob Perry, Oliver Thiel, and Elena Severina

Introduction 1
Genesis and purpose of this book 3
Theoretical basis for the book 4
Structure of the book 5
References 9

PART 1
Explaining 11

2 “One potato, two potatoes …”: mathematics in an outdoor setting 13

Elin Reikerås, Trude Hoel, Børge Iversen, and Aljoscha Jakob Jegodtka

Vignette 13
Reflective questions 17
Commentary 1 17
Commentary 2 20
Commentary 3: domain-specific learning in a play-based outdoor setting: a German perspective 22
PART 2
Counting

3 “Even bigger than this world”: young children thinking about numbers through their mathematical graphics

Elizabeth Carruthers, Alf Coles, and Janet Rose

Vignette 31
Reflective questions 35
Commentary 1: accessing a culture of mathematicians 35
  Counting 35
  Ritual 37
  Conclusion 38
Commentary 2: developing a community of mathematical enquiry 38
  The beginning of the nursery day 39
  Attached teaching 39
  Developing a learning community 39
  Children’s mathematical graphics 40
  Sustained periods of reflective community enquiry 41
Commentary 3: acculturation and enculturation in mathematical understanding – children’s and adults’ “cultural capital” 42
References 45

4 “Let’s roll the dice”: exploring amounts, counting, transcoding, and the investigation of invariance and variance

Aljoscha Jakob Jegodtka, Lilith Schechner, and Kelly Johnston

Vignette 48
Reflective questions 50
Commentary 1 50
  The German context: mathematics in the early years 50
  In the vignette, we see a high-quality addressing of mathematics in kindergarten 52
Commentary 2 53
Commentary 3 55
  The Australian context – mathematics and early learning 55
  Relation to the vignette 57
References 58
PART 3
Measuring

5 Harry’s line work
   Bob Perry, Sue Dockett, Elspeth Harley, Sonya Joseph,
   Marie Hage, and Oliver Thiel

Vignette 63
Reflective questions 66
Commentary 1 66
Curricular and pedagogical contexts and frameworks used in early childhood
   mathematics education in South Australia 66
   Bishop’s six fundamental mathematical activities 68
   Links to curriculum 68
   Potential follow-up activities 69
Commentary 2 70
   Introduction 70
   Players in the vignette 70
   Prior to the experiences recorded in the vignette 71
   Bishop’s six fundamental activities 71
   Post the experience recorded in the vignette 72
   Summary 72
Commentary 3 73
   The didactical sequence of teaching magnitudes in Germany 73
   Young children’s understanding of magnitudes and measuring 74
   The Norwegian case 75
   Possibilities for further experiences 76
   Summary 76
References 76

6 “It’s a lot of work”: a tailor’s measuring tape in the doll’s house
   Maria Figueiredo, Luís Menezes, Helena Gomes, Ana Patrícia Martins,
   António Ribeiro, Myriam Marchese, Isabel Soares, and Lynne Zhang

Vignette 79
Reflective questions 81
Commentary 1 81
   Early childhood mathematics education in Portugal: perspectives from the
      curricular, pedagogical, and research landscape 81
   The tailor’s measuring tape in the doll’s house: two concurring lenses of analysis 83
   Possible steps forward 84
Commentary 2 85
   Contextualisation 85
PART 4
Locating  

7 Fantastic Mr Fox  
Anne Hj. Nakken, Camilla N. Justnes, Oda Bjerknes, and Simone Dunekacke

Vignette 97
Reflective questions 100
Commentary 1 101
Early childhood mathematics education in Norway 101
Noticing the mathematical potential and enriching play 101
Problem-solving 102
Locating 103
Representations 103
Summary 104
Commentary 2 104
Introduction 104
Guiding the children towards new experiences 104
Mathematics and language are linked 105
Different ways of solving problems 105
Pedagogical awareness 106
Summary 107
Commentary 3 107
Mathematics education in German kindergarten 107
Relation to the vignette of Fantastic Mr Fox 109
Summary 110
References 110

8 Ghost stairs and a ghost tree  
Elena Severina, Andreas Lade, and Zoi Nikiforidou

Vignette 114
Selecting photographs 114
Designing pages 116
PART 5
Designing

9 Building bridges between maths and arts
Lucía Casal de la Fuente, Carol Gillanders, Rosa María Vicente Álvarez, and Camilla N. Justnes

Vignette
Reflective questions
Commentary 1
Commentary 2
Commentary 3
References

10 Geometry learning of children in digital activities
Ahmet Sami Konca, Sema Baydilli, and Elena Severina

Vignette
Reflective questions
Commentary 1: geometry learning of children in digital activities
Objectives
Developmental capability
Culture
Digital activity
Summary
Commentary 2
Commentary 3
References

PART 6
Playing

11 “This is the safe. It has a number and no one else knows it”: playing with mathematics
Maulfry Worthington, Elizabeth Carruthers, and Lone Hattingh

Vignette
Contents

Reflective questions 169
Commentary 1: pretend play as a context for mathematics 169
  Culture and mathematics 171
  Mathematics in pretend play 171
  Playing in mathematics 172
  Degrees of freedom 173
  Levels of involvement 174
Commentary 2: the mathematics teacher as key player 174
  The teacher 175
  The teacher as researcher 175
  Resources 176
  Availability of graphic materials 177
  Modelling 177
  No limits on mathematics learning 177
Commentary 3: a perspective from Denmark 178
  The materiality of play 179
  Shared voices: reflections on playing, learning and mathematics 180
References 182

PART 7
Conclusion 187

12 Reaping the benefits of reflexive research and practice in early childhood mathematics education: continuing the conversation 189
  Oliver Thiel, Elena Severina, and Bob Perry

Innovative pedagogy 189
Reflexive practice 191
The Experiential Learning Cycle 192
  Experiencing: the activity phase 193
  Sharing: exchanging reactions and observations 194
  Processing: discussing patterns and dynamics 195
  Generalisation: developing real-world principles 196
  Applying: planning effective use of learning 198
Applications for early childhood educators 198
Applications for teacher educators 199
Future research directions 199
References 200

Index 203
CONTRIBUTORS

Rosa María Vicente Álvarez, Doctor in Pedagogy, collaborates as a researcher in proposals on technology didactic materials and music education. An important contribution is the implementation and development of the “MusicChild” project directed by Dr Pieridou Skoutell. She participates in seminars and conferences about the evaluation of music materials and developing teaching around Europe. Her experience as a preschool, primary, and music teacher provides an extensive background for her work as a teacher at the University of Vigo. She organized the International Symposium of Music Education and Didactic Materials (Santiago de Compostela, 2019).

Sema Baydilli graduated in early childhood from Middle East Technical University. She has worked for eleven years: four years as an early childhood teacher; two years as an educational specialist; and five years as a principal. She has also participated in training programmes, guiding teacher education programmes, and administrative and management processes among the tasks required by the Ministry of Education as well as IB-PYP programmes with teachers. During her working experience, she also wrote early childhood mathematics books for children.

Oda Bjerknes is a kindergarten teacher with several years’ experience teaching in traditional Norwegian kindergartens. Currently, she works as a maths and science teacher in Espira-parken, a facilitated maths and science centre for five-year-old children. In addition, she previously worked part-time with the Norwegian National Center for Mathematics in Education, assisting in the development of activities for the kindergarten department.

Elizabeth Carruthers is currently finishing her doctorate at the University of Bristol. She is the most recent head of Redcliffe Nursery School and National
Teaching School in Bristol, England. Her research interests are early mathematical graphics, teachers as researchers, and early years leadership. She has co-authored two books and writes for teachers and with teachers. Her work on mathematical graphics along with her co-author, Maulfrj Worthington, was recognised by the English Department of Education, Williams Review of Early Years and Primary Mathematics.

Lucía Casal de la Fuente is a singer, psychopedagogue, and researcher at the Department of Pedagogy and Didactics of the University of Santiago de Compostela. Her training and expertise are focused on best practices in vocal education and singing. She has completed several international research stays to delve into singing therapy, childhood, and innovation and equity in education. She coordinates “Voces Ledas,” a psychopedagogical project on vocal education and singing.

Alf Coles is Associate Professor of mathematics education at the University of Bristol, School of Education. He collaborates widely, and his research focus is on the early learning of number, the professional learning of teachers (with a particular interest in practitioner research) and, most recently, on how the mathematics curriculum (in school and higher education) can adapt to both the local concerns of communities (such as pollution) and to global issues (such as climate change). Alf currently co-edits the journal Research in Mathematics Education.

Sue Dockett is Emeritus Professor, Charles Sturt University, Australia. While recently retired from university life, Sue remains an active researcher in the field of early childhood education. Sue has been a long-time advocate for – and of – the importance of recognizing and responding to young children’s perspectives. She maintains this position in her current work with children, families, and educators in explorations of transitions to school, children’s play, and learning.

Simone Dunekacke is Professor at the Freie Universität Berlin, Germany, for Early Childhood Education Research. Her research interest is on early childhood teacher’s professional competence, especially in the domain of early mathematics education. Her research addresses early childhood pre- and in-service teachers and focuses on action-related aspects of professional competence.

Maria Figueiredo is Associate Professor of Educational Sciences, Childhood Education, at the School of Education of the Polytechnic of Viseu, and a researcher at the CI&DEI/IPV. She has been Secretary-General of the European Educational Research Association since 2016. Her work is developed primarily in early childhood and primary school teacher education and social pedagogy. She has researched participatory pedagogies in early childhood education with a strong focus on children’s rights and family involvement.
Carol Gillanders. Since the year 2000 Carol has taught music education at the Faculty of Education of the University of Santiago de Compostela. She is a member of the Educational Technology Research Group directed by Ms Beatriz Cebreiro and has worked in various European projects. Her research interests include service learning, interdisciplinary arts projects, and information and communications technology. She is co-author of a guide for teachers for accompanying songs as well as several articles related to music education.

Helena Gomes is Associate Professor of Mathematics at the School of Education of the Polytechnic of Viseu, and a researcher at the CIDMA/UA. She develops her research activity in the area of mathematics, specifically in algebraic graph theory, and about learning mathematics in early childhood education and in basic education. She has collaborated with the Ministry of Education in the preparation of support materials for the early years and in the evaluation and certification of mathematics textbooks for basic education.

Marie Hage was involved in early childhood education for 40 years as a teacher and director before retiring in 2018. She has been involved in numerous research projects around numeracy, the Reggio Emilia philosophy, and nature education. Marie values giving children uninterrupted time to interact with their environments, to hypothesise, observe, and critique their discoveries. Through observing and documenting children as they play, she has gained a deep understanding of how children best learn.

Elspeth Harley has worked in early childhood education since the late 1960s as a preschool teacher, university lecturer, curriculum adviser, and writer and research associate. She has a background in the arts and drama. Elspeth is committed to promoting the importance of play in the curriculum and in supporting educators to observe and document children’s understandings of numeracy demonstrated in their play. Now retired, she mentors early childhood teachers in preschool and the first years of school settings and is a visiting storyteller and player at her local school and preschool.

Lone Hattingh is Senior Lecturer and Award Leader for MA Education: Early Childhood Studies at Bath Spa University. Her research interests include early literacy with a particular focus on the ways in which children engage with materials to make meaning. Lone is co-leader of REaCH, Bath Spa University’s Centre of Research in Early Childhood, and is currently leading a project about perspectives of outdoor play which draws on case studies in England, Denmark, and the United States.

Trude Hoel is Associate Professor at the National Centre for Reading Education and Research at the University of Stavanger, Norway. She has taken part in several interdisciplinary projects in Norwegian kindergartens, about exciting things like
woodlice and potatoes. Her research concerns early literacy, language learning, and children’s book reading in print and on screen.

**Børge Iversen** is currently working as a preschool teacher in a municipal kindergarten in Trondheim, Norway. He has a master’s degree in early childhood education and graduated as a pedagogista from the Reggio Emilia institute in Stockholm.

**Aljoscha Jegodtka** is currently teaching and researching at the IUBH – International University of Applied Sciences, Berlin. His research focuses on early mathematical education and the development of pedagogical professionalism among early childhood educators. At the same time, he carries out advanced training for early childhood teachers in the field of early mathematical education.

**Kelly Johnston** is a lecturer with the Department of Educational Studies at Macquarie University. She teaches across undergraduate and postgraduate units, specialising in mathematics, science, and technology in early childhood and primary school. Previously, Kelly was engaged in a wide range of roles across the early childhood education sector. This included working as an early childhood teacher and service director in Australia and New Zealand, teaching primary school in the UK, and working in early childhood licensing and accreditation at both a state and federal level in Australia. Kelly’s research focuses on: exploring technology and digital citizenship for young children; mathematical language, thinking, and learning for infants and toddlers; and young children’s learning in museum spaces.

**Sonya Joseph** has worked in early years education for 26 years. She has a bachelor of teaching – junior primary, and is a co-coordinator and educator at Faith Lutheran College, Early Learning Centre in the Barossa Valley. She is interested in the interconnectedness of relationship, environment, and learning and aspires to the principles of Reggio Emilia.

**Camilla Normann Justnes** is Assistant Professor at the Norwegian Centre for Mathematics Education, Norwegian University of Science and Education, Norway. Her research and development work focuses on mathematical discussions and inclusion in kindergarten and lower primary school. Justnes works foremost with developing resources for professional development in mathematics with and for kindergarten teachers, with a particular interest in building a positive view of mathematics for children and their teachers.

**Ahmet Sami Konca** is Assistant Professor at Erciyes University, Turkey. He has a bachelor’s degree in elementary mathematics education, and a master’s degree and a PhD in early childhood education. His main interests are early mathematics education; digital technologies in early childhood education, both in home settings and in classroom settings. He focuses on children’s interactions with digital technologies, digital play, and technology integration into early mathematics education.
**Andreas Lade** is a young teacher in a Norwegian kindergarten with a bachelor's in language, texts and mathematics from the Western Norway University of Applied Sciences. He was one of three kindergarten student teachers who participated in the research project “Photography and image processing in kindergarten as a stimulating activity for mathematics learning”, which directly led to his involvement in this book writing process. He used parts of the research project to further his bachelor's thesis on a similar yet different theme: “How does the kindergarten use the Technological Pedagogical Content Knowledge model (TPCK) to map the staff’s knowledge on information and communication technologies (ICT)? He's currently employed as a kindergarten teacher at a private kindergarten in Bergen, Norway. He strives to lay the best path for the children’s current and future education.

**Myriam Marchese** has a master’s degree in early childhood and primary education. During her studies, she researched how children gave meaning to measurement in an early childhood setting. For three years, she has been working as an early childhood education teacher in the Associação de Beneficência Luso Alemã (ABLA) in Lisbon.

**Ana Patrícia Martins** is Associate Professor of mathematics at the School of Education of the Polytechnic of Viseu and a researcher at the CI&DEI/IPV and at the CIUHCT/FCUL. She holds a PhD in history and philosophy of sciences, a master's degree in mathematics teaching, and she graduated in mathematics. Her research interests include didactic of mathematics, history of mathematics, and history of science.

**Luís Menezes** is Professor at the Higher School of Education of Viseu, Portugal. His main research interests are mathematics teaching practices. In particular, he became interested in the role of communication in mathematics learning. Over 30 years of teaching and research, he has collaborated with the Ministry of Education, having participated in the design of national programmes of mathematics for basic education. In the last decade, he has a growing interest in childhood mathematics education.

**Anne Hj. Nakken** works both as an assistant professor at The Norwegian Centre for Mathematics Education, and as a preschool teacher. Her interests include developing, using, and evaluating research-based resources for preschool teachers, educational technology, transition from preschool to school, mathematics curriculum, and professional development. Anne has over the past 15 years worked on various research and implementation projects focused on improving preschool mathematics teaching and learning.

**Zoi Nikiforidou** is a senior lecturer in early childhood at Liverpool Hope University. Her research interests relate to aspects of cognition, pedagogy, and risk.
Zoi is a member of the OMEP UK executive committee and a co-convenor of the EECERA Special Interest Group *Holistic Wellbeing*.

**Bob Perry** has recently retired after 45 years of university teaching and research. He is Emeritus Professor at Charles Sturt University, Australia and Director, Peridot Education Pty Ltd. In conjunction with Sue Dockett, he continues his research, consultancy, and publication in early childhood mathematics education; educational transitions, with particular emphasis on transition to primary school; researching with children; and evaluation of educational programmes. Bob is co-chair of the EECERA Special Interest Group *Mathematics 0–8 Years*.

**Elin Reikerås** is Professor in early childhood education and Centre leader of FILIORUM – Centre for Research in Early Childhood Education, at the University of Stavanger, Norway. She has completed research on young children’s mathematical development over the last 30 years and published a range of articles and books on the theme. She also led the interdisciplinary longitudinal Stavanger project “The Learning Child,” which followed over 1,000 children’s development from when they were two and a half years old until they turned ten years old.

**António Ribeiro** is Associate Professor of mathematics at the School of Education of the Polytechnic Institute of Viseu. He is a member of CI&DEI/IPV and develops his research activity in the area of didactics of mathematics with a particular interest in the role of teaching resources in the process of teaching and learning mathematics in the first and second cycles of basic education.

**Janet Rose** is currently principal of Norland College, a specialist early years higher education institution. Prior to this, she worked at several universities leading undergraduate and postgraduate early years degree programmes, becoming an associate professor and reader in education at Bath Spa University. She also worked with Elizabeth Carruthers in developing a postgraduate certificate in early years mathematics for specialist leaders to support early years mathematics in order to raise achievement in this subject.

**Lilith Schechner** studied part-time early childhood inclusive education (BA) at the Fulda University of Applied Sciences and worked at the “Kinderhort Gropiusstadt” in Berlin, where she is still employed as a state-recognized childhood educator and childhood educator for language.

**Elena Severina** is Associate Professor at Western Norway University of Applied Sciences with a PhD in physics and mathematics, and a postgraduate certificate in education. Elena teaches mathematics education courses to pre-service preschool, primary, and secondary school teachers and supervises bachelor’s and master’s students. Her research interests are in early childhood mathematics, and include...
critical thinking and argumentation, embodied cognition, spatial reasoning, child-centred digital photography activities, ICT, and multilingual classrooms.

Isabel Soares has been an early childhood teacher for over 35 years in Viseu. For many years, she has welcomed student teachers in her centre, supporting their practicum. She currently works in the Agrupamento de Escolas Infante D. Henrique. In her long professional career with children, she has always valued sensibility and respect as central dimensions of her practice.

Oliver Thiel is Associate Professor at Queen Maud University College in Trondheim, Norway. He has taught early childhood mathematics education in Germany and Norway for over 20 years. His research interests are teachers’ attitudes and beliefs about mathematics and children’s mathematical competence. Oliver is convenor of the EECERA Special Interest Group Mathematics Birth to Eight Years. He is editor of the EECERJ Special Issue “Innovative Approaches in Early Childhood Mathematics” and the German teacher journal Mathematik differenziert (Mathematics differentiated).

Maulfry Worthington taught for almost 30 years in the three- to seven-year age range, and has lectured in early childhood education, and primary and early years mathematics at the undergraduate and master’s level in England. Her research interests include semiotics, graphicacy, children’s mathematics, cultural learning, language acquisition, and pretend play. Her PhD (VU University, Amsterdam) investigates the natural history of signs in young children’s mathematical inscriptions. Maulfry has published extensively, both individually and with Elizabeth Carruthers, and together they founded the international Children’s Mathematics Network.

Lynne Zhang is an early childhood educator, researcher, and founder of Happy Future Connections Children and Family Research Centre which is a prestigious early childhood research, consultancy, and training institute in the social and cultural context for practitioners, parents, researchers, and policymakers in China. She is chief editor of the China Early Childhood Home & Community Learning Report and Playful Mathematics books, and developed early childhood mathematical courses, activities, and materials based on Chinese traditional culture. Lynne is Country Coordinator (China) of EECERA.
FOREWORD

Introduction to the EECERA Book Series

Tony Bertram and Chris Pascal

Underpinning aspirations

This timely, scholarly, and highly readable book edited by Oliver Thiel, Elena Severina, and Bob Perry entitled Mathematics in Early Childhood provides the eighth book in an innovative book series generated by the European Early Childhood Education Research Association (EECERA). The EECERA Book Series entitled Towards an Ethical Praxis in Early Childhood, offers an innovative and exemplary vehicle for the international early childhood sector to develop transformative pedagogy which demonstrates effective integrated praxis. The EECERA Book Series is designed to complement and link with the European Early Childhood Education Research Journal (EECERJ), which is primarily a worldwide academic platform for publishing research according to the highest international standards of scholarship. The EECERA Book Series aims to highlight pedagogic praxis in order to demonstrate how this knowledge can be used to develop and improve the quality of early education and care services to young children and their families.

Pedagogic approach

The approach taken in the book series will not be a linear one, but rather a praxeological one focused on praxis, meaning a focus on pedagogic action impregnated in theory and supported by a belief system. It is this fusion of practice, theoretical perspectives, ethics, and research which we term “Ethical Praxis.” This fusion is embodied in all EECERA research and development activity, but we anticipate the book series will have a stronger focus on the development of
pedagogic praxis and policy. In addition to offering a forum for plural, integrated pedagogic praxis, the series will offer a strong model of praxeological processes that will secure deep improvements in the educational experience of children and families, of professionals and researchers across international early childhood services.

The book series acknowledges pedagogy as a branch of professional/practical knowledge which is constructed in situated action in dialogue with theories and research and with beliefs (values and principles). Pedagogy is seen as an “ambiguous” space, not of one-between-two (theory and practice) but as one-between-three (actions, theories, and beliefs) in an interactive, constantly renewed triangulation. Convening beliefs, values, and principles; analysing practices; and dialoguing with several branches of knowledge (philosophy, history, anthropology, psychology, sociology, amongst others) constitutes the triangular movement of the creation of pedagogy. Pedagogy is thus based on praxis, in other words, action based on theory and sustained by belief systems. Contrary to other branches of knowledge which are identified by the definition of areas with well-defined frontiers, the pedagogical branch of knowledge is created in the ambiguity of a space which is aware of the frontiers but does not delimit them because its essence is in integration.

**Praxeological intentions**

There is a growing body of practitioner and practice-focused research which is reflected in the push at national and international levels to integrate research and analysis skills into the professional skill set of all early childhood practitioners. This is a reflection of the growing professionalism of the early childhood sector and its increased status internationally. The development of higher-order professional standards and increased accountability are reflective of these international trends as the status and importance of early education in the success of educational systems is acknowledged.

Each book in the series is designed to have the following praxeological features:

- strongly and transparently positioned in the sociocultural context of the authors
- practice or policy in dialogue with research, ethics, and with conceptual/theoretical perspectives
- topical and timely, focusing on key issues and new knowledge
- provocative, groundbreaking, innovative
- critical, dialogic, reflexive
- Eurocentric, giving voice to Europe’s traditions and innovations but open to global contributions
- open, polyphonic, prismatic
- plural, multidisciplinary, multimethod
- praxeological, with a concern for power, values, and ethics; praxis; and a focus on action research, the learning community, and reflexive practitioners
- views early childhood pedagogy as a field in itself, not as applied psychology
- concerned with social justice, equity, diversity, and transformation
• concerned with professionalism and quality improvement
• working for a social science of the social
• NOT designed as a textbook for practice but as a text for professional and practice/policy development

This eighth book in our praxis series exemplifies these underpinning philosophies, pedagogical ethics, and scholarly intentions beautifully. We believe it is topical and much needed for these challenging times when educational practices for young children are coming under increasing pressures, focusing on key issues and new knowledge in the teaching and learning of early mathematics. The book is also provocative and critical, encouraging and opening polyphonic and multinational dialogues about our thinking and actions in developing high-quality early childhood services internationally.
Introduction

As we move into a new decade, it is clear that mathematics, and, in particular, early childhood mathematics, is in the international spotlight. The recent release of the 2018 Programme for International Student Assessment (PISA) (OECD, 2019) has forced many countries to reconsider their curriculum, learning, and teaching approaches as results have either stagnated or declined. There are now many studies which seem to show that early childhood mathematics achievement is a strong predictor of success in future school mathematics, other school subjects, and life itself (Carmichael, MacDonald, & McFarland-Piazza, 2014; Claessens & Engel, 2013; Duncan et al., 2007; Geary et al., 2013). As a result, across the globe, there has been much encouragement for early childhood professionals in both prior-to-school and school settings to engage with their children in mathematics learning, with one aim being to ensure that the children’s standards of achievement are higher by the time they meet the first national or international assessment of their careers. However, a recent paper (Watts et al., 2018) has suggested that some of the earlier estimates of the impact of early mathematics interventions on later school mathematics success need to be treated with caution and may have overstated the case.

Taken together, these results lead us to make two primary conclusions. First, correlational approaches to questions regarding longitudinal achievement patterns should be approached with great caution. Second, early learning does not appear to be an “inoculation” that necessarily produces later achievement gains, and consequently, theories regarding skill-building processes probably require some amount of revision.

(Watts et al., 2018, pp. 550–551)
While this later evidence does not deny the importance of early childhood mathematics education for future achievement, it does suggest also that the value mathematics has for children in the present and how children might experience mathematics in their early childhood years be considered. In order to achieve this, early childhood educators, researchers, and policymakers are urged not only to reflect on their practice but also introduce the notion of reflexivity to their thinking. For many, however, the distinction between “reflection” and “reflexivity” is unclear. Bolton (2010, pp. 13–14) helps with two descriptions.

**Reflection** is learning and developing through examining what we think happened on any occasion, and how we think others perceived the event and us, opening our practice to scrutiny by others, and studying data and texts from the wider sphere.

...  

**Reflexivity** is finding strategies to question our own attitudes, thought processes, values, assumptions, prejudices and habitual actions, to strive to understand our complex roles in relation to others. To be reflexive is to examine, for example, how we – seemingly unwittingly – are involved in creating social or professional structures counter to our own values. (Italics in original)

Early childhood educators are urged to move beyond reflection and towards reflexivity; to consider what they and others believe and value; and to undertake their practice on the basis of such reflexivity. They should ask not only “What happened?” but also “Why did it happen?” and “What can I do about it?” Such a move is particularly required in the area of early childhood mathematics education, which has often been minimised in early childhood settings in spite of its long history through luminaries such as Fröbel (Fröbel & Lilley, 1967) and Montessori (1912).

A strong influence on early childhood mathematics education over recent years is the advent of the neoliberal political and advocacy juggernaut known as STEM (Science, Technology, Engineering, and Mathematics). While the STEM movement has made mathematics visible, there is a danger that mathematics will be seen only to be the “servant” of science, technology, and engineering and that all mathematics will need to be drawn from these other disciplines or apply to them. Such an approach does fit well with early childhood approaches such as relevance, experiential learning, and play, but also has the potential to reduce realisation of the uniqueness of mathematics, particularly mathematical thinking, with a nature and approach which demand respect in its own right (Devlin, 2012; Hardy, 1940). In early childhood, mathematics provides opportunities for challenge, investigation, discovery, and sustained shared thinking (Siraj-Blatchford, 2007) that are not restricted to utilitarian applications, but also stimulate creative and innovative thinking in both young children and their educators (Shen & Edwards, 2017). Experiences with mathematics in the early years develop thinking and reasoning for young children’s present and future (Katz, 2010).
Current research perspectives indicate that mathematics is important in the here and now of early childhood as well as into the future; that there are some consequences of early childhood mathematics education for later learning, although the scope of these is under question; that early childhood educators are urged to adopt reflexive practices; and that while mathematics is very important in young children’s lives, it also has a role in developing particular forms of knowledge and thinking in its own right, both for the present and for the future. How can all of this be achieved in early childhood education settings in the best possible way for the child? One response has been what Moss (2014) has dubbed “schoolification” and which he has critiqued in the following way:

“schoolification,” an expressive term for primary schooling taking over early childhood institutions in a colonising manner (OECD, 2006, p. 62), leading to a school-like approach to the organisation of early childhood provision, the adoption of “the content and methods of the primary school” with a “detrimental effect on young children’s learning” (OECD, 2001, p. 129), and “neglect of other important areas of early learning and development” (p. 42). While mathematics, language and science matter, the question is how best to work with them in early childhood education; while the problem is how to avoid them contributing to further schoolification by the spread of crude and oversimplified educational approaches that are at odds with the learning strategies of young children and that end up doing more harm than good.

(p. 37)

In the field of early childhood mathematics education, this book seeks to help answer the call for approaches which work well but do not bring associated detrimental effects.

**Genesis and purpose of this book**

This book is the product of the Special Interest Group (SIG) on *Mathematics Birth to Eight Years* within the *European Early Childhood Education Research Association* (EECERA). (Information on both EECERA and the SIG can be obtained from www.eecera.org/). The EECERA SIG *Mathematics Birth to Eight Years* aims to coordinate and disseminate international research on the discourse in the emerging early childhood mathematics education field. It creates a space for shared thinking and for creating synergies between participants from a wide range of professional and scientific contexts to encourage a clearer articulation and understanding of early childhood pedagogy, policy and practice in relation to mathematics.

*(EECERA, 2020)*
In 2017, at the EECERA conference in Bologna, Italy, SIG members determined that they would undertake a single research project across numerous jurisdictions, the results of which could be used to generate a book in the EECERA series *Towards an Ethical Praxis in Early Childhood*. After a nomination, review, and selection process, teams of early childhood researchers and educators generated vignettes about young children’s mathematical experiences. These vignettes described real, rather than hypothetical, snapshots of children’s experiences and provided rich examples of mathematics in early childhood settings. They were jointly developed by at least one early childhood researcher and at least one early childhood educator, bringing research and practice together. For each of these vignettes, three commentaries were written — the first by one of the researchers involved with the vignette; the second by an early childhood educator involved in writing the vignette; and the third by an early childhood mathematics education researcher working in a jurisdiction different from the origin of the vignette. Each chapter in the book consists of a vignette and the three associated commentaries. A follow-up face-to-face meeting of many of the authors was held at the EECERA conference in Thessaloniki in 2019 where decisions were made about the final content and nature of the book, and presentations were made about the overall research project.

The final product of the project focuses on children’s mathematics learning through play and other pedagogical approaches and is based on research, policy, and practice from several different contexts around the world. It is a fusion of research and practice emanating from a consistent approach to analysis and critiquing vignettes of practice which encourages both researchers and practitioners to work together to consider their own practice and values associated with this practice reflexively. The book has met the overall objective of the project: to present innovative, research-based, and practical pedagogic praxis with the aim of enhancing young children’s experiences in, and learning of, mathematics.

**Theoretical basis for the book**

The research project and each chapter of this book are based on an analysis by Alan Bishop (1988a, 1988b) of mathematics as a cultural pursuit, and his theory of mathematical enculturation. Bishop (1988b) has argued that there are six fundamental mathematical activities which “are both universal, in that they appear to be carried out by every cultural group ever studied, and also necessary and sufficient for the development of mathematical knowledge” (p. 182).

Mathematics, as cultural knowledge, derives from humans engaging in these six universal activities in a sustained, and conscious manner. The activities can either be performed in a mutually exclusive way or, perhaps more significantly, by interacting together, as in “playing with numbers.”

*(Bishop, 1988b, p. 183)*
Each of the chapters in this book is assigned to one of Bishop’s activities, but all of them highlight interactions among them. These activities and the explanations offered by Bishop (1988b) are listed in Table 1.1.

### Table 1.1 Bishop’s six fundamental mathematical activities (derived from Bishop, 1988b, pp. 182–183)

<table>
<thead>
<tr>
<th>Fundamental mathematical activity</th>
<th>Bishop’s descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting</td>
<td>The use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names.</td>
</tr>
<tr>
<td>Locating</td>
<td>Exploring one’s spatial environment and conceptualising and symbolising that environment, with models, diagrams, drawings, words, or other means.</td>
</tr>
<tr>
<td>Measuring</td>
<td>Quantifying qualities for the purposes of comparison and ordering, using objects or tokens as measuring devices with associated units or “measure-words.”</td>
</tr>
<tr>
<td>Designing</td>
<td>Creating a shape or design for an object or for any part of one’s spatial environment. It may involve making the object, as a “mental template,” or symbolising it in some conventionalised way.</td>
</tr>
<tr>
<td>Playing</td>
<td>Devising, and engaging in, games and pastimes, with more or less formalised rules that all players must abide by.</td>
</tr>
<tr>
<td>Explaining</td>
<td>Finding ways to account for the existence of phenomena, be they religious, animistic, or scientific.</td>
</tr>
</tbody>
</table>

Each of the chapters in this book is assigned to one of Bishop’s activities, but all of them highlight interactions among them. These activities and the explanations offered by Bishop (1988b) are listed in Table 1.1.

### Structure of the book

There are twelve chapters in this book with a total of 37 authors (some of whom have more than one contribution) who are either working in and/or have cultural backgrounds in eleven countries. The first chapter – this chapter – has been written to provide the focus for the contributed chapters that follow. Chapters 2 to 11 contain vignettes and commentaries and are organised according to Bishop’s fundamental mathematical activities.

The structure of each of Chapters 2 to 11 is the same, commencing with a vignette and followed by the three commentaries. Following the vignette and before the commentaries there is a set of Reflective Questions designed so that readers are urged to bring their own experiences, values, and beliefs to the vignette. These can then be compared with and critiqued against the commentaries. Each of the chapters is meant to be read as a whole, but separately from the other chapters. It is not anticipated that readers would read from front to back cover in one sweep. Rather, this is a book to be “dipped into” on the basis of a Bishop activity, interest in a particular topic, or perhaps on the cultural genesis of the authors. To this end, brief details are supplied in Table 1.2 for each of Chapters 2 to 11.
<table>
<thead>
<tr>
<th>Chapter title</th>
<th>Vignette topic</th>
<th>Key Bishop activity</th>
<th>Genesis of commentary authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. “One potato, two potatoes …” – Mathematics in an outdoor setting</td>
<td>This vignette shows how an outdoor activity, a potato harvest, gives rich possibilities for facilitating play-based mathematical activities adapted to children at different ages.</td>
<td>Explaining</td>
<td>Norway Norway Germany</td>
</tr>
<tr>
<td>3. “Even bigger than this world” – young children thinking about numbers through their mathematical graphics</td>
<td>In this vignette, the teacher has put out a range of maths equipment following on from children’s previous enquiries about infinity. For example, there is a range of markers, clipboards, an abacus, metre rulers, and a number grid 1–100.</td>
<td>Counting</td>
<td>United Kingdom United Kingdom United Kingdom/ South Africa</td>
</tr>
<tr>
<td>4. “Let’s roll the dice” – Exploring amounts, counting, transcoding, and investigating invariance and variance</td>
<td>This vignette takes place in a German kindergarten. The children take turns throwing a dice, counting the number of pips on its faces, naming the respective numeral, and gathering that number of toy blocks.</td>
<td>Counting</td>
<td>Germany Germany Australia</td>
</tr>
<tr>
<td>5. Harry’s Line Work</td>
<td>The vignette commences when Harry – a 4-year-old Australian preschool boy – seeks out Danette, the administrative officer for the preschool, who is at her desk. She is using a ruler to measure and Harry takes some interest in this, asking Danette what she is doing and how she is using the instrument.</td>
<td>Measuring</td>
<td>Australia Australia Norway/Germany</td>
</tr>
<tr>
<td>Chapter title</td>
<td>Vignette topic</td>
<td>Key Bishop activity</td>
<td>Genesis of commentary authors</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------</td>
<td>---------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>6. “It’s a lot of work” – A tailor’s measuring tape in the dollhouse</td>
<td>There is a group of 26 children mainly from Roma families and low socio-economic background in a public early childhood centre in a small Portuguese city. During free playtime, a small group of children noticed a measuring tape that was left in the dollhouse with no explanation given.</td>
<td>Measuring</td>
<td>Portugal</td>
</tr>
<tr>
<td>7. Fantastic Mr Fox</td>
<td>Three children aged 4 to 5 years play in the sandbox, and Emma, the kindergarten teacher, observes them. The children have listened to the book “The Fantastic Mr Fox” by Roald Dahl. They are afterwards occupied by foxes in their free play.</td>
<td>Locating</td>
<td>Norway</td>
</tr>
<tr>
<td>8. Ghost stairs and a ghost tree</td>
<td>The story comes from a project on photography and mathematics in a Norwegian kindergarten. Three groups of four 5-year-old children took pictures of what they found beautiful in the outdoor environment, selected photos, and used them to design a photobook on a computer with a touchscreen.</td>
<td>Locating</td>
<td>Norway/ Russia</td>
</tr>
</tbody>
</table>

(continued)
<table>
<thead>
<tr>
<th>Chapter title</th>
<th>Vignette topic</th>
<th>Key Bishop activity</th>
<th>Genesis of commentary authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. Building bridges between maths and arts</td>
<td>A workshop for a single-room rural preschool in Spain with six children aged between 3 and 6 years was offered. The aim of the workshop was to link maths and music with the ultimate goal of promoting mathematical thinking in early childhood through music.</td>
<td>Designing</td>
<td>Spain Spain Norway</td>
</tr>
<tr>
<td>10. Geometry learning of children in digital activities</td>
<td>The vignette takes place in an early childhood setting in the suburb of a small Turkish town. The digital activity consisted of drawing a locomotive that was composed of triangles, rectangles, squares, and circles.</td>
<td>Designing</td>
<td>Turkey Turkey Norway/Russia</td>
</tr>
<tr>
<td>11. “This is the safe. It has a number and no one else knows it” – Playing with mathematics</td>
<td>The vignette focuses on an episode of pretend play in which the boys’ collaborative dialogue grew from their personal interest in security safes. During the course of their play, the boys made reference to many aspects of number and quantities, culminating in communicating ideas through emergent mathematical inscriptions.</td>
<td>Playing</td>
<td>United Kingdom United Kingdom United Kingdom/Denmark</td>
</tr>
</tbody>
</table>
The final chapter provides a synthesis of previous chapters, highlighting the connections between research and the professional practice of observing children, writing vignettes, and analysing them in terms of Bishop's fundamental activities. It uses the research data reported in each of the chapters to introduce an experiential learning framework which can be used to enhance young children's mathematical learning. The editors argue for the importance of reflexive practice as educators engage with, listen to, and respond to young children’s early mathematical activities. The chapter concludes with a range of possible ways to continue research and praxis conversations around appropriate methods and methodologies for supporting early childhood mathematics teaching and learning.

The editors of Mathematics in Early Childhood: Research, Reflexive Practice and Innovative Pedagogy have enjoyed their role in creating this book and they commend it to anyone who is working in, or interested in, the field of early childhood mathematics education. It is a different book, bringing together a group of authors through a common objective: to enhance reflexivity in mathematics learning and teaching for young children. The book is innovative in design and rich in detail. Hopefully, it will stimulate early childhood educators and researchers to continue striving for the very best in mathematics education for their children and themselves.

Notes

1 Nursery schools in England are a unique early years’ provision that are government funded. They have a tradition dating back to the 1920s, rooted in the work of Margaret Macmillan and Susan Isaacs (Edgington, 1998).
2 The title “National Teaching School” is awarded to a school that has been judged outstanding and has the capacity to be a centre of professional development, research, and initial teacher training: these schools are funded for their work.
3 Ros Sutherland’s untimely death cut short her involvement in this writing, but she has been present for me in writing this commentary. Ros’s passion was for social justice (Sutherland, 2007), and I know she would have enjoyed commenting on this vignette, perhaps seeing in it an opening up of access to the power of mathematical practices and culture.
1 The GDR had curricula for kindergartens for a long time. This was different in the Federal Republic of Germany. After reunification in 1990, there were no kindergarten curricula in Germany for about 15 years. As late as 2004, the federal states started to introduce educational plans.
1 In Turkish, while daire means circle, yuvarlak is used to imply a round shape. Moreover, yuvarlak tends to be used instead of daire for circle in daily language. However, as the meanings of yuvarlak and daire are not actually the same, Asya ensured that the children correctly used daire when naming a circle.
2 Here, in alliance with the vignette and the other commentaries, I do not count squared shapes as rectangular to be in coherence with the children’s level in van Hiele’s model.
1 Nursery schools in England are a unique early years’ provision that are government funded. They have a tradition dating back to the 1920s, rooted in the work of Margaret Macmillan and Susan Isaacs (Giardiello, 2013).
Both teachers and early years’ practitioners work in the nursery school, but for brevity, the word “teacher” is used throughout.

Regrettably, Isaac’s mathematical inscription is not sufficiently clear to reproduce it here.

The nursery’s approach to mathematics developed from work by Carruthers and Worthington.

Prior to four years of age, some children will attend nursery schools, whilst others may attend preschool playgroups or childminders.

The teaching of “synthetic phonics” especially, has had a significant impact on all aspects of teaching and learning in the Foundation Stage in England. In turn, this has exerted new pressures on early years mathematics.

The four rules listed here and the explanation of each are taken from van Oers (2013, pp. 191–192).

In this instance, technology appears to be a dominant interest (see, for example, Pirani and Hussain (2019), who explore the value of technology for learning in the early years).

The term “hot spot” relates to biodiversity and describes a region that is a significant reservoir of biodiverse species. The diversity (of children’s mathematical thinking, talk, conceptual knowledge, and graphical representations) is exemplified in this and other chapters, thereby endorsing it as a mathematical (and play) hot spot.

The title “National Teaching School” is awarded to a school which has previously developed research and is a centre of initial teacher training; these schools are funded for their work.

References


References


Kuhne, C., O’Carroll, S., Comrie, B., & Hickman, R. (2012). Much more than counting: supporting mathematics development between birth and five years. Cape Town: The Schools Development Unit (UCT) and Wordworks.


References


the development of children’s mathematical competencies: results of an exploratory triangulated videographic study].


Buys, K., & de Moor, E. (2008). Domain description measurement. In M. van den Heuvel-Panhuizen (Ed.), *Young children learn measurement and geometry: A learning-teaching trajectory with intermediate attainment targets for the lower grades in primary school* (pp. 15–36). Rotterdam: Brill.


Føsker, L. I. R. (2012). Grip rommet! Barns utvikling av romforståelse og barnehagelærerens systematiske arbeid med det [Catch the space! Children’s development of spatial understanding and kindergarten teachers’ systematical work with it]. In T. Fosse (Ed.),
References

Rom for matematikk – I barnehagen [Space for mathematics – in the kindergarten], (pp. 61–89). Bergen: Caspar Forlag.


Elia, I. (2018) Observing the use of gestures in young children's geometric thinking. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank, & C. Benz (Eds.), Contemporary research and...
perspectives on early childhood mathematics education (pp. 159–182). ICME-13 Monographs. Cham: Springer.


References


References


Sinclair, N., & Moss, J. (2012). The more it changes, the more it becomes the same: The development of the routine of shape identification in dynamic geometry environments. International Journal of Education Research, 51 & 52, 28–44.


References


References


