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ASYMPTOTICS, NONPARAMETRICS, AND TIME SERIES

edited by

SUBIR GHOSH

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Riverside, California

A Tribute to Madan Lal Puri

Marcel Dekker, Inc.
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Madan Lal Puri
Asymptotic and nonparametric methods are widely used in statistics; time series is an important area of statistics. Many researchers have contributed and others are working to develop these subjects. As a result, we observe a profusion of research. This reference book is a collection of articles describing some of the recent developments and surveying some important topics. The book is a tribute to Professor Madan Lal Puri, who has contributed vigorously to asymptotic and nonparametric methods and their application in time series. This collection of articles is a present to Professor Puri for his 70th birthday to celebrate his contributions, leadership, and dedication to our profession. This is a collection not just by his friends but by the world leaders in their special research areas. The topics covered are broader than the title describes. Parametric, semiparametric, frequentist, bayesian, bootstrap, adaptive, univariate and multivariate methods, Markov chain models, and many others are also discussed in this book. All the articles have been refereed and are in general expository. The book should be of value to students, instructors, and researchers at colleges and universities, as well as in businesses, industries, and government organizations.
The following individuals were truly outstanding for their cooperation and help in reviewing the articles: Shun-ichi Amari, Gopal K. Basak, Jan Beran, Johanne F. Böhme, Dennis D. Boos, Jack Cuzick, Rainer Dahlhaus, Clive W. J. Granger, Cindy Greenwood, Wouter Den Hann, Nancy Heckman, Lajos Horvath, Irene Hueter, Harry Hurd, Wesley O. Johnson, Jerry H. Klotz, Eric D. Kolaczyk, Masao Kondo Johannes Ledolter, Ernst Linder, Olive Linton, Richard Lockhart, Bani K. Mallick, Marianthi Markatou, Michael A. Martin, Jean Meloche, Serena Ng, Dimitris N. Politis, Gregory C. Reinsel, Moshe Shaked, David Stoffer, Arnold J. Stromberg, Winfried Stute, Robert L. Taylor, Ram C. Tiwari, Stephen G. Walker, Edward C. Waymire, Granville T. Wilson, Wayne A. Woodward, Daming Xu, and G. Alastair Young. I am grateful to all our distinguished reviewers.

My deep appreciation and heartfelt thanks go to our renowned contributors, who I hope forgive me for not telling them in advance about some details regarding this book. But then, a surprise for Professor Madan Lal Puri and our contributors will uplift our spirits and stimulate us to contribute more to our society.

My sincere thanks go to Russell Dekker, Maria Allegra, and others at Marcel Dekker, Inc. I would like to thank my wife, Susnata, and our daughter, Malancha, for their support and understanding of my efforts in completing this project.

Subir Ghosh
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Madan Lal Puri: Life and Contributions of a Mathematical Statistician

Madan Lal Puri was born in Sialkot (then in India, now in Pakistan) on February 20, 1929. In 1947, when India gained her independence and Pakistan was created, his family migrated to Delhi as refugees. He received a B.A. degree in 1948 and an M.A. degree in 1950, both in mathematics, from Panjab University in India. From January 1951 to August 1957, he served as a Lecturer in Mathematics in different colleges of Panjab University.

In September 1957, he came to the United States as an instructor and a graduate student in mathematics at the University of Colorado in Boulder. In September 1958, he moved to the University of California at Berkeley as a research assistant in the Department of Statistics and then received his Ph.D. in statistics in 1962. His dissertation was in the area of nonparametric inference under the guidance of Professor Erich L. Lehmann. As a world center of statistics in the 1950s and 1960s University of California at Berkeley hosted a number of world renowned experts in probability and statis-

The publications of Madan Lal Puri are given in the Appendix.
Madan Lal Puri

Statistics: Cox, Cramer, Doob, Feller, Hoeffding, Hotelling, Kiefer, Robbins, Kendall, Rao, and Wolfowitz, to name only a few. The list of faculty members during this golden period includes, among others, the prominent statisticians Blackwell, LeCam, Lehman, Loeve, Neyman, and Scheffe. Madan received his statistics education in this exciting environment.

In 1962, Dr. Madan L. Puri joined the Courant Institute of Mathematical Sciences at New York University as an Assistant Professor and became an Associate Professor in 1965. He joined Indiana University at Bloomington in 1968 as a full Professor of Mathematics and remains there to this day.

Professor Puri is one of the most versatile and prolific researchers in mathematical statistics. His research areas include nonparametric statistics, order statistics, limit theory under mixing, time series, splines, tests of normality, generalized inverses of matrices and related topics, stochastic processes, statistics of directional data, random sets, fuzzy sets and fuzzy measures, among others. His fundamental contributions in developing rank-based methods and precise evaluation of the standard procedures, asymptotic expansions of distributions of rank statistics, as well as large deviation results concerning them, span various areas, such as analysis of variance, analysis of covariance, multivariate analysis, and time series. His in-depth analysis resulted in many pioneering research contributions in prominent journals which have substantial impact on current research.

Professor Puri has done joint work with many researchers from different countries. His numerous joint contributions with Professor P. K. Sen in the 1960s and 1970s on rank-based procedures and their asymptotic properties are greatly valued in our profession. Researchers in many other disciplines use these statistical procedures in their everyday work. This joint collaboration resulted in two advanced books, *Nonparametric Methods in Multivariate Analysis of Variance* and *Nonparametric Methods in General Linear Models*, which are still the leading books on these topics. In the 1980s and 1990s, his pioneering contributions with Professor M. Hallin resulted in new rank-based methods for time series analysis. His many joint contributions on convergence rates with Professors R. N. Bhattacharya, M. Harel, S. Ralescu, M. Seoh, and T. J. Wu; on generalized inverses with Professor S. K. Mitra; on fuzzy sets and measures with Professor D. A. Ralescu; on invariance principles for stochastic processes with Professors M. Denker, G. Haiman, and H. Harel; and on rank-based methods in the analysis of designed experiments with Professor E. Brunner are just a few of his many such accomplishments.

Until the early 1980s the theory and practice of rank-based inference was essentially limited to analysis with independent observations. This limitation on independent or, at least, exchangeable observations was more or less inherent in rank-based inference. The papers published between 1985 and
1994 by Hallin and Puri, as well as Hallin, Ingenbleek, and Puri, present detailed rank-based methods for the analysis of the popular autoregressive-moving average (ARMA) and other models. This much-needed development was made possible through the use of a new type of rank statistics, called the \textit{serial linear rank statistics}, introduced by Hallin and Puri.

Professor Puri was the Alexander von Humboldt Guest Professor at the University of Göttingen in West Germany during 1974–75 and Guest Professor at many other universities in Germany, with German National Science Foundation grants. He was a Distinguished Visitor at the London School of Economics and Political Science, and Visiting Professor at the University of Auckland in New Zealand, the Universities of Bern and Basel in Switzerland, the University of New South Wales in Australia, the University of Goteborg and Chalmers University of Technology in Sweden, and Université des Sciences et Technologies de Lille France. He was invited to lecture at the Japanese Society for the Promotion of Science in 1971. He has been an invited speaker as well as plenary speaker at many international conferences all over the world.

Professor Puri has received numerous honors and awards. He is an elected member of the International Statistical Institute, and a Fellow of the Institute of Mathematical Statistics, the American Statistical Association, and the Royal Statistical Society. In 1975, he was honored with the D.Sc. degree from Panjab University in India. He twice received the Senior U.S. Scientist Award from the Alexander von Humboldt Foundation, in 1974 and 1983. In 1974, he was honored by the government of the Federal Republic of Germany, “In recognition of past achievements in research and teaching.” In 1984, he received the best paper award from the Seventh European Meeting on Cybernetics and Systems Research, Vienna, Austria. In 1991, he received the Rothrock Faculty Teaching Award in recognition of outstanding teaching in the Department of Mathematics of Indiana University. He was ranked as the fourth most prolific author in 1997 and the ninth most in 1993 in top statistical journals in the world.

Professor Puri served on organizing committees of many international conferences in addition to those of the Institute of Mathematical Statistics and the American Statistical Association. He also served as Editor-in-Chief of the \textit{Journal of Statistical Planning and Inference} during 1984–1988.

Professor Puri has directed 16 Ph.D. dissertations. Most of his former Ph.D. students are in research and teaching positions at good universities. A few of them hold responsible positions in industry.

Professor Puri is truly an international academician and a peripatetic scholar who works with missionary zeal. Many scientists from different countries visit him regularly and do research with him while staying at his home. He is a caring colleague with warmest affection, an international host,
a persuasive communicator, a dedicated as well as an outstanding teacher, and a versatile statistician whose work continues to inspire the scientific community.

With great pleasure, pride, and admiration, we dedicate this book in honor of Professor Madan Lal Puri on his 70th birthday. The age of seventy is a time of liberation, a time to realize that there is more to do, more to see, and more reasons to be around for the people who really appreciate you. Madan, our best wishes to you on this happy occasion and in years to come.

Subir Ghosh
George G. Roussas
1

Some Examples of Empirical Fourier Analysis in Scientific Problems

DAVID R. BRILLINGER  University of California, Berkeley, California

"One can FT anything—often meaningfully."

J. W. Tukey

1. INTRODUCTION

As a concept and as a tool, the Fourier transform is pervasive in applied mathematics, computing, mathematics, probability and statistics as well as in substantive sciences such as chemistry, geophysics and physics. This chapter presents a review of such applications and then four personal analyses of scientific data based on Fourier transforms. Specific points made include: Fourier analysis is conceptually simple, its concepts often have direct physical interpretations, useful statistical properties are available, and there are various interesting connections between the mathematical and physical concepts.

By Fourier analysis is meant the study of spaces and functions, making substantial use of sine and cosine functions. The subject has a long and glorious history. In particular, fundamental work has been carried out by both mathematicians and applied scientists. Fourier analysis remains of interest to mathematicians because generalizations seem inexhaustible and because there are continual surprises. Classic works by mathematicians
include: Wiener (1933), Bochner (1959, 1960) and Zygmund (1968). These particular authors are concerned with functions on the line or on a general Euclidian space. Works on extensions to general groups include: Loomis (1953), Rudin (1962), Hewitt and Ross (1963), Kadznelson (1976). More recent books are Terras (1988) and Köner (1989), the former particularly addressing the nonabelian case, the latter presenting a variety of historical examples and essays on specific topics.

In contrast, the Fourier transform is of interest to statisticians because it proves inordinately useful in the analysis of data and eases the development of various theoretical results. Noteworthy contributions to statistics have been made by Slutsky (1934), Cramér (1942), Good (1958), Yaglom (1961), Tukey (1963), Hannan (1965, 1966), Priestley (1965), Bloomfield (1976), Diaconis (1988, 1989). Slutsky developed some of the statistical properties of the Fourier transform of a stretch of time series values. Cramér set down a Fourier representation (see Sec. 4.1) for stationary processes. The representation admitted many extensions and made transparent the effect of a variety of operations on processes. Good and Tukey indicated how the transform could be computed recursively and hence quickly in certain circumstances. Yaglom extended the domain of application to processes defined on compact and locally compact groups. Hannan considered problems for other groups than Yaglom and presented material relevant to practical applications. Priestley provided a frequency domain representation to describe nonstationary processes. Bloomfield made complicated results available to a bread audience. Diaconis considered symmetric and permutation groups and the analysis of ordered data.

Particular uses of the empirical Fourier transform include: the development of estimates of finite dimensional parameters appearing in time series models (Whittle (1952), Dzhaparidze (1986), Feuerverger (1990)), the assessment of goodness of fit of models (Feigin and Heathcote (1976)), and the deconvolution of random measurements (Fan (1992)). Fourier analysis has a special place amongst the tools of statistics for the concepts often have their own physical existence.

There are special computational, mathematical and statistical properties and surprises associated with the Fourier transform. These include: the central limit theorems for the stationary case with approximate independence at particular frequencies, the existence of fast Fourier transforms, (Good (1958), Tukey (1963), Cooley and Tukey (1965), Rockmore (1990)) the need for convergence factors, the ideas of aliasing.

Section 2 concerns some particular physical situations. Section 3 contains pertinent background from analysis. Section 4 contains stochastic background. Section 5 presents analyses of four data sets from the natural sciences and the author's experience. The examples highlight
Examples of Empirical Fourier Analysis

approximation, shrinkage estimation, the method of stationary phase, central limit theorems and uncertainty estimation. The first example, concerning crystallographic data, involves the empirical representation of a basic function on the plane by an expansion in sines and cosines. This makes sense because of periodicities inherent in the crystal structure. The example also involves shrinkage of the coefficients of the expansion in order to obtain improved estimates. The second analysis is of a record of an earthquake that took place in Siberia as recorded at Uppsala, Sweden. The oscillatory character of the data may be understood heuristically via the method of stationary phase, to be described below. A model of the transmission medium is constructed and model assessment carried out by a sliding or dynamic Fourier analysis. This last may be viewed as a form of wavelet analysis. The third analysis, concerned with nuclear magnetic resonance (NMR) spectroscopy, employs Fourier analysis to obtain physical insight into the behavior of an input-output system, and then makes use of cross-spectral analysis to estimate the transfer function of the system. The periodogram of the residuals is employed to assess the fit. The final example involves both wavelet and Fourier analysis. It is concerned with the question of whether a microtubule moves steadily or via jumps. The Fourier analysis is employed in this case to obtain uncertainty estimates in the presence of stationary noise. Section 6 contains conclusions and indicates open problems.

2. SOME PHYSICAL EXAMPLES OF FOURIER ANALYSIS

Cycles, periods, and resonances have long been noted in scientific discussions of astronomy, vibrations, oceanography, sound, light and crystallography amongst other fields. In technology oscillations occur often for example in telephone, radio, TV and laser engineering. Natural operations occur commonly that correspond with linear and time invariant systems as defined in Section 3 below. These are the eigenoperations of Fourier analysis.

Fourier analysis is sometimes tied specifically to the physics of a problem. For example Bazin et al. (1986) physically demonstrate the operations/concepts of translation, linearity, similarity, convolution and Parseval's theorem for the Fourier transform via diffraction experiments with laser light. The Fourier transform here is formed via a lens. See Goodman (1968) Shankar et al. (1982), Glaeser (1985) for a discussion of the optics involved.

An important example arises in radio astronomy. Suppose there is an array of receivers. Suppose there is a small incoherent source, at great distance, producing a plane travelling wave. If \( Y(x,y,t) \) denotes the radio field measurement made at time \( t \) on a telescope located at position \( (x,y) \), then
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